Numerical Modeling of Stratified Shallow Flows: Applications to Aquatic Ecosystems

Marica Pelanti[1,2], Marie-Odile Bristeau[1] and Jacques Sainte-Marie[1,2]

[1]INRIA Paris-Rocquencourt, [2]EDF R&D

Joint work with

- E. Audusse (Univ. Paris XIII), B. Perthame (Univ. Paris VI, INRIA)
- N. Goutal, M.-J. Salençon (EDF R&D)

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Outline

1. Motivations
2. A variable-density multilayer Saint-Venant model
3. A Reservoir model based on Navier–Stokes: OPHÉLIE©EDF
4. Numerical simulations
5. Perspectives

Modeling of complex free surface environmental flows

Aquatic ecosystems: lakes, estuarine waters, lagoons,...

Related applications: water quality, sustainability of ecosystems.


Lake of Geneva (*Lac Leman*)


Étang de Berre

- Important vertical stratification (temperature, salinity);
- Column structure result of complex 3D forcing mechanisms:
  - Natural: rivers inflows, wind, solar radiation,...
  - Man-made: inflow/outflow in hydraulic reservoirs, pollutant spills,...
- Bio-dynamics primarily controlled by thermo-chemical status.

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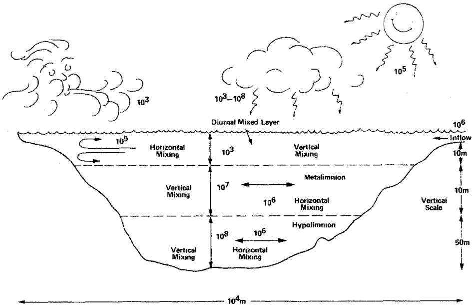
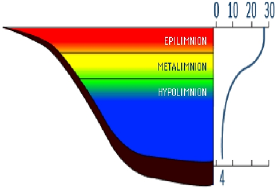
Typical lake stratification

wind rain solar radiation





Lake stratification with estimates of energy transfer time scales [s]
[J. Imberger, *Hydrobiologia* 125, 7–29, 1985]

Temperature vertical profile [°C]
(http://waterontheweb.org)

A Multilayer Saint-Venant Model with Variable Density

- Saint-Venant models

Main assumption: $\varepsilon = H_0/L_0 \ll 1$ (shallow flow)

- Classical Saint-Venant equations

- Uniform density and vertically uniform velocity;
- Hydrostatic pressure (vertical velocity $= 0$).

Multilayer Saint-Venant models

- Allow non-uniform velocity along the vertical.
  - ◦ Immiscible layers [Ovsyannikov 79, Vreugdenhil 79, Parés et al. 00,01,03 ...]
  - ◦ Multilayer model with mass exchange
    [Audusse–Bristeau–Perthame–Sainte-Marie, 2009]
    - Allows fluid circulation between layers.

Goal: Introduce variable density description via the multilayer strategy.

Variable-Density Multilayer Saint-Venant System

Hydrostatic Euler equations with $\rho = \rho(T)$, T = active tracer

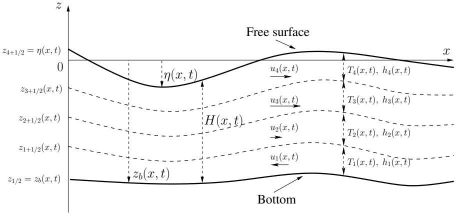
$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho w)}{\partial z} = 0\,,$$

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2)}{\partial x} + \frac{\partial (\rho u w)}{\partial z} + \frac{\partial p}{\partial x} = 0\,,$$

$$\frac{\partial p}{\partial z} = -\rho g\,,$$

$$\frac{\partial (\rho T)}{\partial t} + \frac{\partial (\rho u T)}{\partial x} + \frac{\partial (\rho w T)}{\partial z} = 0\,.$$

+ B.C.



$h_\alpha = l_\alpha H,\ \sum_{\alpha=1}^{N} l_\alpha = 1$

$u(x,z,t) \approx \sum_{\alpha=1}^{N} 1_{z\in h_\alpha}(z) u_\alpha(x,t)$

$T(x,z,t) \approx \sum_{\alpha=1}^{N} 1_{z\in h_\alpha}(z) T_\alpha(x,t)$

Variable-Density Multilayer Saint-Venant System

Unknowns: $H, u_\alpha, T_\alpha,\ \alpha = 1, \ldots, N$

$$\frac{\partial}{\partial t}\left(\sum_{\alpha=1}^N \rho_\alpha h_\alpha\right) + \frac{\partial}{\partial x}\left(\sum_{\alpha=1}^N \rho_\alpha h_\alpha u_\alpha\right) = 0\,,$$

$$\frac{\partial}{\partial t}(\rho_\alpha h_\alpha u_\alpha) + \frac{\partial}{\partial x}\left(\rho_\alpha h_\alpha u_\alpha^2 + h_\alpha p_\alpha\right)$$
$$= u_{\alpha+1/2} G_{\alpha+1/2} - u_{\alpha-1/2} G_{\alpha-1/2} + \frac{\partial z_{\alpha+1/2}}{\partial x} p_{\alpha+1/2} - \frac{\partial z_{\alpha-1/2}}{\partial x} p_{\alpha-1/2}\,,$$

$$\frac{\partial}{\partial t}(\rho_\alpha h_\alpha T_\alpha) + \frac{\partial}{\partial x}\left(\rho_\alpha h_\alpha u_\alpha T_\alpha\right)$$
$$= T_{\alpha+1/2} G_{\alpha+1/2} - T_{\alpha-1/2} G_{\alpha-1/2}\,, \quad \alpha = 1, \ldots, N.$$

$$h_\alpha = l_\alpha H, \quad z_{\alpha+1/2}(x,t) = z_b(x) + \sum_{j=1}^\alpha h_j(x,t)\,,$$
$$p_\alpha = \frac{g}{2}\rho_\alpha h_\alpha + g\sum_{j=\alpha+1}^N \rho_j h_j\,, \quad p_{\alpha+1/2} = g\sum_{j=\alpha+1}^N \rho_j h_j\,,$$
$$G_{\alpha+1/2} = \frac{\partial}{\partial t}\left(\sum_{j=1}^\alpha \rho_j h_j\right) + \frac{\partial}{\partial x}\left(\sum_{j=1}^\alpha \rho_j h_j u_j\right), \quad \rho_\alpha = \rho(T_\alpha)\,.$$

- Hyperbolicity ? Typically hyperbolic in practice.

Numerical Method

System of $2N+1$ equations in $X = (H, \{T_\alpha, q_\alpha\}_{1\leq\alpha\leq N})$, $q_\alpha = \rho_\alpha h_\alpha u_\alpha$:

$$\frac{\partial}{\partial t} W(X) + \frac{\partial}{\partial x} f(X) = \underset{\uparrow G_{\alpha\pm 1/2}}{S_G(X, \partial_{t,x} X)} + \underset{\uparrow p_{\alpha\pm 1/2}}{S_p(X)} \ .$$

- Conservative portion and S_G: Finite Volume Kinetic Scheme [Perthame–Simeoni, 2001]
  - Positivity preserving;
  - Avoids eigenvalues computation.
- Source S_p: Hydrostatic Reconstruction Method [Audusse et al., 2004]
  - Well-balancing of equilibrium states. Particular case at rest:

$$u_\alpha = 0, \quad H + z_b = \text{const.}, \quad \rho_\alpha = \text{const.}, \ \alpha = 1, \ldots, N.$$

Additional implemented features:

◦ 2nd order in space (MUSCL) and time (Heun);

◦ Vertical viscosity for velocity and temperature (implicit method).

OPHÉLIE

Hydrodynamic reservoir model based on Navier–Stokes.
[M.-J. Salençon, J.Y. Simonot, 1989] ©EDF

2D laterally averaged hydrostatic incompressible Navier–Stokes eqs.

- Rigid lid hypothesis: $\frac{\partial z_s}{\partial x} = \frac{\partial z_s}{\partial y} = 0$.

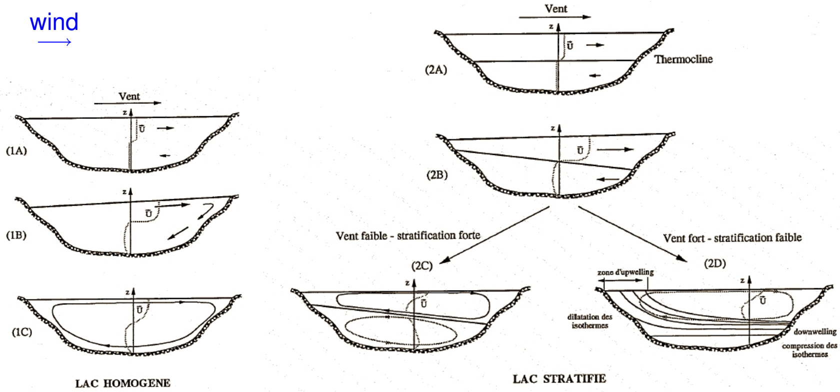
$$\frac{\partial(Bu)}{\partial x} + \frac{\partial(Bw)}{\partial z} = 0, \qquad B(x,z) = \text{lake width},$$
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = \frac{1}{B}\frac{\partial}{\partial x}\left(\nu_x B\frac{\partial u}{\partial x}\right) + \frac{1}{B}\frac{\partial}{\partial z}\left(\nu_z B\frac{\partial u}{\partial z}\right) - \frac{\partial p}{\partial x} - g\frac{u|u|}{BC^2},$$
$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z} = \frac{1}{B}\frac{\partial}{\partial x}\left(D_x B\frac{\partial T}{\partial x}\right) + \frac{1}{B}\frac{\partial}{\partial z}\left(D_z B\frac{\partial T}{\partial z}\right) - \frac{1}{\rho_0 C_p}\frac{\partial Q}{\partial z},$$
$$p = p_s + g\rho_0(z_s - z) - \kappa g\rho_0\int_z^{z_s}(T-T_0)^2 dz, \;\; \rho(T) = \rho_0(1-\kappa(T-T_0))^2.$$

⋆ Includes surface thermodynamics.
⋆ Rigid lid vs. free-surface: much less restrictive CFL constraint. (BUT limited range of applications.)

Model developed for specific application: Lake of Sainte-Croix.

Lake response to wind stress

wind
$\longrightarrow$



Homogeneous lake

Stratified lake

M.-J. Salençon and J.-M. Thébault, *Modélisation d'écosystème lacustre*, 1997.

(After J. Imberger, 1979, 1987).

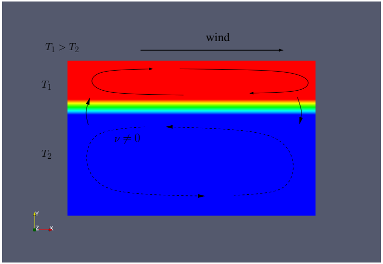
Numerical Tests - Multilayer Saint-Venant model

Wind stress modeling

Additional source term: $S_W = C_{10}\frac{\rho_a}{\rho_W}|V_W|V_W$ applied on surface layer N.

V_W = wind velocity, C_{10} = wind stress coefficient for wind at 10 m.
ρ_a = air density, ρ_w = water density.

T = temperature, $\rho(T) = \rho_0(1-\kappa(T-T_0))^2$, $T_0 = 4\,°$C.



$T_1 = 20\,°$C , $T_2 = 7\,°$C, $\nu = 0.03$ m/s^2.

Test 1. $V_W = 10$ m/s

$\Delta H_{\text{meta}} = 2.5$ m.

(a) Passive tracer. ρ = const.

(b) $\rho = \rho(T)$.

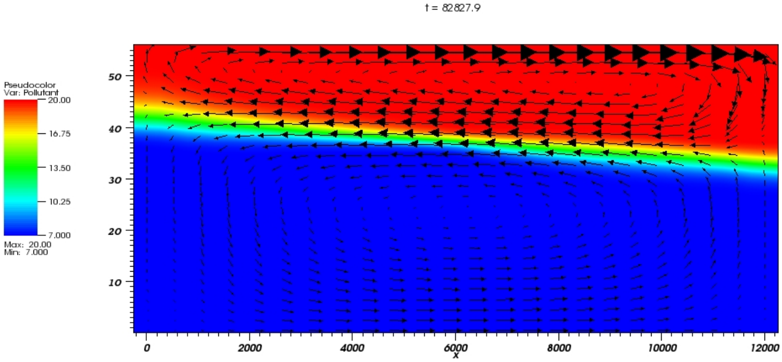
Num. layers = 56, x-grid cells = 24.

CFL = 0.8.

Test 1

$V_{\mathrm{W}} = 10$ m/s, $\Delta H_{\mathrm{meta}} = 2.5$ m.

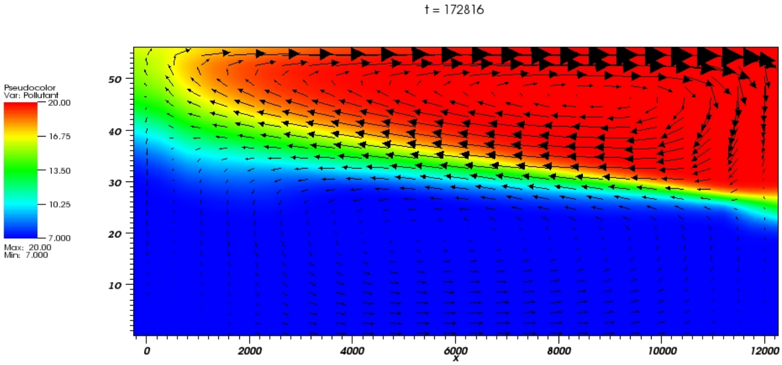
Uplifting of the thermocline until steady position.



Test 2: Upwelling

$V_{\mathrm{W}} = 15$ m/s, $\Delta H_{\mathrm{meta}} = 20$ m.

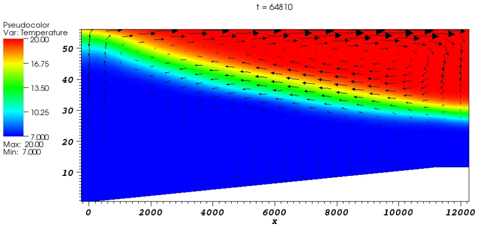
Upwelling of the middle layer (metalimnion).

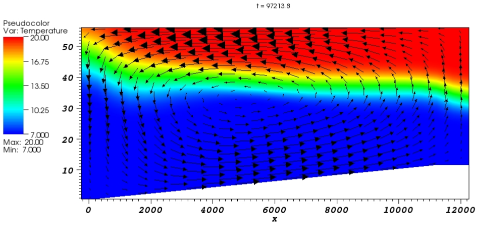


Test 3: Variable topography

$V_{\mathrm{W}} = 15$ m/s, $\Delta H_{\mathrm{meta}} = 20$ m.

Simulation with wind stopped after 18 hours.

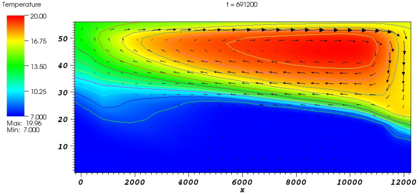




Limitations of the hydrostatic assumption

- Model lacks description of unstable stratification.
  Static equilibrium stable if $\frac{\partial \rho}{\partial z} \leq 0$.

Test 2 at later times:



Whereas physically:
cold water front propagation
with vertical isotherms.
(cf. results OPHÉLIE)

Non-hydrostatic terms need to be included in the model:

$$\frac{\partial}{\partial t}(\rho w) + \frac{\partial}{\partial x}(\rho u w) + \frac{\partial}{\partial z}(\rho w^2) + \frac{\partial p}{\partial z} = -\rho g.$$

- Preliminary results with modeling of $\frac{\partial}{\partial z}(\rho w^2)$ (Exp. Rayleigh).

OPHÉLIE: Stabilization of temperature profile by mixing.

Current and future work

Current studies

- Variable-density multilayer Saint-Venant model
  - Non-hydrostatic terms.
  - Surface thermodynamics.

Prospected work

- Comparison multilayer S.-V. / OPHÉLIE (time scale $\sim$ days).
- Coupling hydrodynamics with biological processes.
- OPHÉLIE application to realistic environment (Lac de Grangent).