

fd_explicit_cir1d_zbo

Input parameters:

- Time StepNumber M

Output parameters:

- Price

The stochastic differential equation representing the short rate is given by

$$dr_t = k(\theta - r_t)dt + \sigma \sqrt{r_t}dW(t)$$

The price of the zero-coupon bond is solution of the following PDE

$$u_t + \frac{1}{2}\sigma^2 r u_{rr} + [k(\theta - r)]u_r - ru = 0, u(r, T, T) = 1$$

that we solve using explicit scheme of Hull-White[1]. The price of the option is obtained solving the same PDE with boundary condition at the maturity of the option T , the price of the Zero Coupon Bond. The pricing procedure is in two steps: in the first step(INITPROBA) we compute probabilities associated to the explicit scheme; this is done simply matching the first and the second moment of the change in r over time step Δt . The branching in the lattice is modified at boundary points $r = r_{min}$ and $r = r_{max}$ to ensure that the probabilities associated with all three branches remain positive. For this purpose Hull-White[1] propose alternative branching procedures in the explicit finite difference method.

The second step is standard dynamic programming backward pricing algorithm.

References

- [1] J.Hull and A.WHITE. Valuing derivative securities using the explicit finite difference method. *Journal of Financial and Quantitative Analysis*, 25:87–100, 1990. [1](#)