

cf_vasicek1d_cap

Output parameters:

- Price

The stochastic differential equation representing the the shor rate is given by

$$dr_t = k(\theta - r_t)dt + \sigma dW(t)$$

The price of the zero-coupon bond is given by

$$P(t, T) = A(t, T)e^{-B(t, T)r(t)}.$$

where

$$A(t, T) = e^{(\theta - \frac{\sigma^2}{2k^2})(B(t, T) - T + t) - \frac{\sigma^2}{4k} B(t, T)^2}$$

and

$$B(t, T) = \frac{1}{k}(1 - e^{-k(T-t)})$$

The price of the European Put with maturity T on Zero-Coupon Bond with maturity ($S > T$) is given by

$$KP(t, T)N(\sigma_p - h) - P(t, S)N(-h)$$

Where N is the cumulative function of the normal law and

$$\sigma_p = \sigma \sqrt{\frac{1 - e^{2k(T-t)}}{2k}} B(T, S)$$

and

$$h = \frac{1}{\sigma_p} \log\left(\frac{P(t, S)}{P(t, T)k}\right) + \frac{\sigma_p}{2}$$

A cap is equivalent to a portfolio of European zero-coupon Put-Options.