

cf_callupin

Reiner-Rubinstein [1] have developed formulas for pricing standard barrier options:

Let

- T = maturity date ($T > t$)
- K = strike price
- L = up barrier
- R = rebate
- x = spot price
- t = pricing date
- σ = volatility
- r = interest rate
- δ = dividend yields
- $\theta = T - t$
- $b = r - \delta$

We set here:

- $A = \phi x e^{-\delta\theta} N(\phi x_1) - \phi K e^{-r\theta} N(\phi x_1 - \phi \sigma \sqrt{\theta})$
- $B = \phi x e^{-\delta\theta} N(\phi x_2) - \phi K e^{-r\theta} N(\phi x_2 - \phi \sigma \sqrt{\theta})$
- $C = \phi x \left(\frac{L}{x}\right)^{2(\mu+1)} e^{-\delta\theta} N(\eta y_1) - \phi K e^{-r\theta} \left(\frac{L}{x}\right)^{2\mu} N(\eta y_1 - \eta \sigma \sqrt{\theta})$
- $D = \phi x \left(\frac{L}{x}\right)^{2(\mu+1)} e^{-\delta\theta} N(\eta y_2) - \phi K e^{-r\theta} \left(\frac{L}{x}\right)^{2\mu} N(\eta y_2 - \eta \sigma \sqrt{\theta})$
- $E = R e^{-r\theta} \left[N(\eta x_2 - \eta \sigma \sqrt{\theta}) - \left(\frac{L}{x}\right)^{2\mu} N(\eta y_2 - \eta \sigma \sqrt{\theta}) \right]$
- $F = R \left(\frac{L}{x}\right)^{\mu+\lambda} N(\eta z) + \left(\frac{L}{x}\right)^{\mu-\lambda} N(\eta z - 2\eta \lambda \sigma \sqrt{\theta})$

and

- $dA = \phi e^{-\delta\theta} N(\phi x_1)$
- $dB = \phi e^{-\delta\theta} N(\phi x_2) + e^{-\delta\theta \frac{n(x_2)}{\sigma\sqrt{\theta}}} (1 - \frac{k}{l})$
- $dC = \frac{1}{x} \phi 2\mu \left(\frac{L}{x}\right)^{2\mu} \left(x e^{-\delta\theta \frac{L^2}{x^2}} N(\eta y_1) - K e^{-r\theta} N(\eta y_1 - \eta\sigma\sqrt{\theta}) - \phi \left(\frac{L}{x}\right)^{2(\mu+1)} e^{-b\theta} N(\eta y_1) \right)$
- $dD = -2\mu \frac{\phi}{x} \left(\frac{L}{x}\right)^{2\mu} \left(s e^{-\delta\theta \frac{L^2}{x^2}} N(\eta y_2) - K e^{-r\theta} N(\eta y_2) \times \right. \\ \left. \times N\left(\eta y_2 - \eta\sigma\sqrt{\theta} - \phi \left(\frac{L}{x}\right)^{2\mu+2} e^{-\delta\theta}\right) N(\eta y_2) - \phi \eta \left(\frac{L}{x}\right)^{2\mu+2} e^{-\delta\theta} \right) \times \\ \left. \times \frac{n(y_2)}{\sigma\sqrt{\theta}} \left(1 - \frac{k}{l}\right) \right)$
- $dE = 2\frac{R}{x} e^{-r\theta} \left(\frac{L}{x}\right)^{2\mu} \left[N(\eta y_2 - \eta\sigma\sqrt{\theta})\mu + \eta \frac{n(\eta y_2 - \sigma\sqrt{\theta})}{\sigma\sqrt{\theta}} \right]$
- $dF = -\frac{R}{x} \left(\frac{L}{x}\right)^{\mu+\lambda} \left[(\mu + \lambda) N(\eta z) + (\mu - \lambda) \left(\frac{L}{x}\right)^{2\lambda} \right] - 2\eta R \left(\frac{L}{x}\right)^{\mu+\lambda} \frac{n(z)}{x\sigma\sqrt{\theta}}$

where

- $x_1 = \frac{\log(x/K)}{\sigma\sqrt{\theta}} + (1 + \mu)\sigma\sqrt{\theta}$ $x_2 = \frac{\log(x/L)}{\sigma\sqrt{\theta}} + (1 + \mu)\sigma\sqrt{\theta}$
- $y_1 = \frac{\log(L^2/xK)}{\sigma\sqrt{\theta}} + (1 + \mu)\sigma\sqrt{\theta}$ $y_2 = \frac{\log(L/x)}{\sigma\sqrt{\theta}} + (1 + \mu)\sigma\sqrt{\theta}$
- $z = \frac{\log(L/x)}{\sigma\sqrt{\theta}} + \lambda\sigma\sqrt{\theta}$
- $\mu = \frac{b - \sigma^2/2}{\sigma^2}$ $\lambda = \sqrt{\mu^2 + \frac{2r}{\sigma^2}}$

and η, ϕ belong to $\{-1, 1\}$ and will be fixed later.

Let

$$M_{t,T} = \sup_{t \leq \tau \leq T} S_\tau \quad \text{and} \quad m_{t,T} = \inf_{t \leq \tau \leq T} S_\tau$$

Up-In Call Option

$$\begin{array}{ll}
\text{PAYOFF} & C_T = \begin{cases} R & \text{if } M_{t,T} < L, \\ (S_T - K)_+ & \text{otherwise.} \end{cases} \\
\text{PRICE} & P(t, x) = \begin{cases} A + E & \text{if } K > L, \\ B - C + D + E & \text{otherwise} \end{cases} \\
\text{DELTA} & \frac{\partial C(t, x)}{\partial x} = \begin{cases} dA + dE & \text{if } K > L, \\ dB - dC + dD + dE & \text{otherwise} \end{cases}
\end{array}$$

and take

$$\eta = -1 \quad \phi = 1$$

References

- [1] E.REINER M.RUBINSTEIN. Breaking down the barriers. *Risk*, 4:28–35, 191. [1](#)