# mc_robbinsmoro_hes

**Input parameters**

- Number of iterations $N$
- Generator type
- Increment $inc$
- Confidence Value
- Volatility of volatility

**Output parameters**

- Price $P$
- Error price $\sigma_P$
- Delta $\delta$
- Error delta $\sigma_{delta}$
- Price Confidence Interval: $ICp$ [Inf Price,Sup Price]
- Delta Confidence Interval: $ICp$ [Inf Delta,Sup Delta]

## Description

Computation of a european option in the Heston stochastic volatility model.

This model is given by,

$$dS_t = (r-q)S_t dt + \sqrt{v_t} S_t dW_t^1,$$
$$dv_t = k(\theta - v_t)dt + \sigma\sqrt{v_t} dW_t^2,$$

where $W^1$ and $W^2$ are two correlated brownian motions with $\langle W^1, W^2\rangle_t = \rho t$, and $k$, $\theta$ and $\sigma$ are constants.
/*The Heston closed formula*/
In the case of a European call option, Heston [1] guessed a solution of the form

$$C(S,v,t) = Se^{-q(T-t)}P_1 - Ke^{-r(T-t)}P_2,$$

by analogy with the Black-Scholes formula. The first term in this formula is the present value of the spot price while the second term is the present value of the strike-price payment.
Using this model, Heston has given a closed form solution to the pricing of a European call option by the caracteristic functions technique.

## MC pricing Algorithm:

/*The price*/
The objective is to compute $V_0 = \mathbb{E}[\phi(S_T)]$ where $(S_t)_{t\leq T}$ is the Heston model.
/*Simulate the discretized underlying*/
Discretizing with an *Euler scheme* leads to

$$S_{T_{i+1}} = S_{T_i}(1 + (r-q)\Delta t + \sqrt{\sigma_i \Delta t} Z_i),$$
$$v_{T_{i+1}} = v_{T_i} + k(\theta - v_{T_i})\Delta t + \sigma\sqrt{\Delta t v_{T_i}}(\rho Z_i + \sqrt{1-\rho^2} Z_{m+i}),$$

where $(Z_i)_{i\geq 1}$ is a sequence of independent Gaussian variables with mean 0 and variance 1. In our implementations we have taken the stochastic input to model to be the single vector $(Z_1,\ldots,Z_{2m})$. In some respects, it might be more natural to think of two separate vectors, each of length $m$. So at each iterations generate a random gaussian vector of size $2\times N$ where $N$ is the total number of MC iterations. Then separate this vector in two vectors af same size $N$ to simulate both the underlying asset and the volatility.
/*Importance sampling*/
In the discretized problem we have to evaluate $\hat{V}_0 = \mathbb{E}[\hat{\phi}(Z)]$ where $Z = (Z_1,\ldots,Z_m)$ is a standard gaussian vector. Using an elementary version of Girsanov theorem leads to the following representation of $\hat{V}_0$:

$$\hat{V}_0 = \mathbb{E}[g(\mu, Z)], \tag{1}$$

with

$$g(\mu, Z) = \hat{\phi}(Z + \mu) e^{-\mu \cdot Z - \frac{1}{2}\|\mu\|^2}, \tag{2}$$

where $\|x\|$ denotes the Euclidean norm of a vector $x \in \mathbb{R}^m$ and $x \cdot y$ is the inner product of two vectors $x, y \in \mathbb{R}^m$.
/*Variance reduction*/
The idea is then to make use of a Robbins and Monro algorithm to assess the optimal $\mu^*$ that minimizes the variance of $g(\mu^*, Z)$.
/*The price computation and confidence interval*/
By rebalancing this optimal $\mu^*$ in the MC computation of the price we reduce the variance by a factor of 5 and more. Finally by the use of the central limit theorem we get a confidence interval with a length equal to the length of the MC standard confidence interval by a factor of 2.5 to 3 and even more for options that are far from the money.

# References

[1] S.L.HESTON. A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies*, 6(2):327–343, 1993. 2