

## fd\_multigrid\_euro\_bs

Input parameters:

- Number of grids  $l$
- TimeStepNumber  $M$
- Theta  $\frac{1}{2} \leq \theta \leq 1$

Output parameters:

- Price
- Delta

We refer to Hackbusch [1] for a detailed presentation of multigrid methods.

**/\*SpaceStepNumber  $N^*$ \*/**

$N = nn(l) + 1$  where  $nn(l)$  calculates the number of points in the grid of level  $l$ .

**/\*Memory Allocation\*/**

**/\*Time Step\*/**

Define the time step  $k = \frac{T}{M}$ .

**/\*Space localisation\*/**

Define the integration domain  $D = [-limit, limit]$  using the probabilistic estimate [there](#).

**/\*Space Step\*/**

Define the space step  $h = \frac{2 * limit}{M}$ .

**/\*Peclet Condition\*/**

If  $|r - \delta|/\sigma^2$  is not small, then a more stable finite difference approximation is used. cf [there](#).

**/\*Rhs factor of theta scheme\*/**

Initialize the matrix  $N$  issued from the  $\theta$ -scheme method in the cases of Dirichlet Boundary conditions. [there](#)

**/\*Terminal value\*/**

After a logarithmic transformation, put the value of the payoff into a vector  $P$  which will be used to save the option value.

**/\*Finite difference Cycle\*/**

At any time step, we have to solve the linear discrete problem of [there](#) which can be written in the following form

$$L^l u^l = f^l \tag{1}$$

(where  $f^l$  denotes *Rhs*).

**/\*Init Rhs\*/**

**/\*Multigrid Method\*/**

We solve the linear discrete problem using the Multigrid method. The multigrid iteration (V-cycle) at level  $l$  for solving [1](#) is defined by the following recursive procedure:

$$v_l \leftarrow MGM(l, v^l, f^l)$$

**Step 1** **/\*factor of theta scheme\*/**

Initialize the matrix  $L^l$  issued from the discretization of the operator  $A$  in the case of Dirichlet Boundary conditions.

Relax 2 times on  $L^l u^l = f^l$  with a given initial guess  $v^l$ .

**Step 2** if  $\Omega^l$  is the coarsest grid ( $l=0$ ) then go to step 4.

Else

$$f^{l-1} \leftarrow I_l^{l-1}(f^l - L^l v^l).$$

$$v^{l-1} \leftarrow 0.$$

$$v^{l-1} \leftarrow MGM(l-1, v^{l-1}, f^{l-1})$$

**Step 3** Correct  $v^l \leftarrow v^l + I_{l-1}^l v^{l-1}$ .

**Step 4** Relax 2 times on  $L^l u^l = f^l$  with initial guess  $v^l$ ,

where  $I_{l-1}^l$  is the linear interpolation operator and  $I_l^{l-1}$  the restriction operator.  $I_{l-1}^l$  is defined by the rule  $I_{l-1}^l v^{l-1} = v^l$  where

$$\begin{aligned} v_{2j}^l &= v_j^{l-1} \\ v_{2j+1}^l &= \frac{1}{2}(v_j^{l-1} + v_{j+1}^{l-1}), \quad 0 \leq j \leq \frac{n}{2} - 1. \end{aligned}$$

The restriction operator is defined by  $I_l^{l-1} v^l = v^{l-1}$ , where

$$v_j^{l-1} = v_{2j}^l.$$

**/\*Price\*/**

**/\*Delta\*/**

**/\*Memory Desallocation\*/**

## References

- [1] Hackbusch, W.: Multi-grid methods and applications. Springer-Verlag. (1985) [1](#)