

fd_fmgh

Input parameters:

- Number of grids l
- TimeStepNumber M
- Theta $\frac{1}{2} \leq \theta \leq 1$
- Epsilon

Output parameters:

- Price
- Delta

We use a multigrid algorithm based on the "Howard algorithm" (policy iteration) [3] and the multigrid method [4]. We refer to Akian [1] for a detailed presentation.

/*StepNumber N */

$N = nn(l) + 1$ where $nn(l)$ calculates the number of points in the grid of level l .

/*Memory Allocation*/

/*Time Step*/

Define the time step $k = \frac{T}{M}$.

/*Space localisation*/

Define the integration domain $D = [-limit, limit]$ using the probabilistic estimate [there](#).

/*Space Step*/

Define the space step $h = \frac{2 * limit}{N}$.

/*Peclet Condition*/

If $|r - \delta| / \sigma^2$ is not small, then a more stable finite difference approximation is used. cf [there](#).

/*Lhs factor of theta scheme*/

Initialize the matrix M^h issued from the discretization of the operator A in the case of Dirichlet Boundary conditions. cf [there](#).

/*Rhs factor of theta scheme*/

Initialize the matrix N issued from the θ -scheme method in the cases of Dirichlet Boundary conditions. [there](#)

/*Terminal value*/

After a logarithmic transformation, put the value of the payoff into a vector P which will be used to save the option value.

/*Finite difference Cycle*/

At any time step, we have to solve the linear complementarity problem cf. [there](#).

/*Init pp and R*/

/*Howard cycle*/

We solve the linear complementarity problem using the Howard algorithm, which consists in constructing a convergent sequence u^p whose limit is u .

Let $\epsilon > 0$ be given.

Step 1 Let u^k be given, we compute $i \rightarrow pp^k[i] = \operatorname{argmin}(M^{pp}u^k(i) - f^{pp}[i])$ where $pp = 0$ or 1 (the domain is divided into 2 regions: the continuation region and the exercise region), M^0 is the matrix M^h issued from the discretization of the operator A , $M^1 = Id$, $f^0 = R$, $f^1 = Obst$.

Step 2 We solve the linear system $M^{pp^k}u = G^{pp^k}$ by the multigrid method. It gives u^{k+1} .

The stopping criteria is

$$\|u^{k+1} - u^k\|_{\infty} < \epsilon. \quad (1)$$

/*Price*/

/*Delta*/

/*Memory Desallocation*/

References

- [1] Akian, M.: Méthodes multigrilles en contrôle stochastique. Thèse de doctorat de l'université Paris 9 Dauphine. (1990) [1](#)
- [2] Hackbusch, W.: Multi-grid methods and applications. Springer-Verlag. (1985)
- [3] Howard, R.A.: Dynamic Programming and Markov Process. (MIT Press. 1960) [1](#)
- [4] Mc Cormick, S.F.: Multigrid methods. SIAM frontiers in applied mathematics. 5 (1987). [1](#)