

Multi-asset Spread Option Pricing

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The authors of [1] give the following second-order boundary approximation method to compute a multi-asset spread option price.

We are interested in computing the following risk-neutral expectation

$$\Pi = e^{-rT} \mathbb{E} \left(\left[S_0(T) - \sum_{k=1}^N S_k(T) - K \right]^+ \right) \quad (1)$$

where the $\log(S_k(T))$ are jointly normally distributed, in [1], both geometric Brownian motion and log-Ornstein-Uhlenbeck models are considered. However, we only implement this method for the GBM model.

We define μ_k and ν_k as follows :

$$\begin{aligned} \mu_k &= \mathbb{E}[\log S_k(T)] \\ \nu_k^2 &= \text{Var}[\log S_k(T)] \end{aligned} \quad (2)$$

under the GBM model, they become:

$$\begin{aligned} \mu_k &= \log S_k(0) + (r - q_k - \sigma_k^2/2)T \\ \nu_k^2 &= \sigma_k^2 T \end{aligned} \quad (3)$$

where q_k denotes S_k 's convenience yield and σ_k its volatility.

The price Π is given by the following formula:

$$\Pi = e^{-rT} \left[e^{\mu_0 + \nu_0^2/2} I_0 - \sum_{k=1}^N e^{\mu_k + \nu_k^2/2} I_k - K I_{N+1} \right] \quad (4)$$

The integrals I_k are approximated at second order as:

$$I_k \approx J_k^0 + J_k^1 - J_k^2/2 \quad (5)$$

The expressions of J_k^i are given in [1] and we don't reproduce them in this document.

The deltas are given by :

$$\begin{aligned} \Delta_0 &= +e^{-q_0 T} I_0 \\ \Delta_k &= -e^{-q_k T} I_k, \quad k = 1 \dots N \end{aligned} \quad (6)$$

Function Usage

`static void ap_spread_li(...)`

The arguments are the following:

- `PnlVect *s0`
Initial price vector $(S_0(0), \dots, S_N(0))$
- `double K`
Strike price K
- `double t`
Maturity T
- `double r`
Risk-free interest rate r
- `PnlVect *divid`
Convenience yield vector (q_0, \dots, q_N)
- `PnlVect *sigma`
Volatility vector $(\sigma_0, \dots, \sigma_N)$
- `PnlVect *delta`
Delta vector $(\delta_0, \dots, \delta_N)$, the values are updated with the function
- `double rho`
The correlation ρ between S_i and S_j , for $i \neq j$
- `int flag_app`
- `double *ptprice`
The pointer to the price computed using the function

References

- [1] Li, M., Deng, S. and Zhou, J. Multi-asset Spread Option Pricing and Hedging
Quantitative Finance, Vol 10-3, 2010