cf_putmin

Let

- T = maturity date $\quad (T > t)$
- K = strike price
- $x1$ = spot1 price
- $x2$ = spot2 price
- t = pricing date
- $\sigma 1$ = volatility1
- $\sigma 2$ = volatility2
- ρ = correlation
- r = interest rate
- $\delta 1$ = dividend1 yields
- $\delta 2$ = dividend2 yields
- $\theta = T - t$

Here, closed formulas due to Johnson and Stulz are presented [1],[2].
We set

- $d = \dfrac{\log \frac{x_1}{x_2} + \left(\delta_2 - \delta_1 + \frac{\sigma^2}{2}\right)\theta}{\sigma\sqrt{\theta}}$
- $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$
- $d_i = \dfrac{\log\left(\frac{x_i}{K}\right) + \left(r - \delta_i + \frac{\sigma_i^2}{2}\right)\theta}{\sigma_1\sqrt{\theta}}, \quad i = 1, 2$
- $\rho_1 = \dfrac{\sigma_1 - \rho\sigma_2}{\sigma}$
- $\rho_2 = \dfrac{\sigma_2 - \rho\sigma_1}{\sigma}$

and M as the cumulative bivariate normal distribution function:

$$M(a,b;\rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{a} \int_{-\infty}^{b} e^{-\frac{x^2-2\rho xy+y^2}{2(1-\rho^2)}} dx dy.$$

Put On Minimum Option

PAYOFF $P_T = (K - \min(S_T^1, S_T^2))_+$

PRICE $P(t, x_1, x_2) = Ke^{-r\theta} - c_0 + c_1$

DELTA $$\frac{\partial P(t, x_1, x_2)}{\partial x_1} = e^{-\delta_1\theta}(1 - N(d)) + e^{-\delta_1\theta}M(d_1, -d; -\rho_1)$$
$$\frac{\partial P(t, x_1, x_2)}{\partial x_2} = e^{-\delta_2\theta}N(d - \sigma) + e^{-\delta_2\theta}M(d_2, d - \sigma\sqrt{\theta}; -\rho_2)$$

where

- $c_0 = x_1 e^{-\delta_1\theta}(1 - N(d)) + x_2 e^{-\delta_2\theta}N(d - \sigma\sqrt{\theta})$
- $c_1 = x_1 e^{-\delta_1\theta}M(d_1, -d, -\rho_1) + x_2 e^{-\delta_2\theta}M(d_2, d - \sigma\sqrt{\theta}; -\rho_2)$
  $-Ke^{-r\theta}M(d_1 - \sigma_1\sqrt{\theta}, d_2 - \sigma_2\sqrt{\theta}; \rho)$

References

[1] H.JOHNSON. Options on the maximum ot the minimum of several assets. *J.Of Finance and Quantitative Analysis*, 22:227–283, 1987. 1

[2] R.STULZ. Options on the minimum or the maximum of two risky assets. *J. of Finance*, 10:161–185, 1992. 1