Title : Static Hedging of Standard Options [1]

Authors : P. Carr and L. Wu

Premia 22

1 Summary of the article

1.1 Spanning options with options

The authors consider the hedging of european options on one asset S in a markovian framework satisfying the following hypothesis;

> **Assumption :** *There exists a deterministic pricing function* $C : \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}^d \to \mathbb{R}$ *such that the price* $C_t(K,T)$ *of a call option at time* t *with maturity* T *and strike* T *on the underlying* S *is equal to* $C(S_t, t, K, T, \theta)$ *where* θ *represents the deterministic model parameters.*

Under this set of assumptions, they derive the following result :

> **Theorem 1:** *Under no arbitrage and the markovian assumption above, the time-t value of a european call option maturing at a fixed time* $T \geq t$ *is related to the time* t*-values of a continuum of european call options at a shorter maturity* $u \in [t, T]$ *by :*

$$\begin{array}{rcl} C(S,t,K,T,\theta) & = & \int_0^{+\infty} w(k) C(S,t,k,u,\theta) dk, \;\; u \in [t,T] \\ where \;\; w(k) & = & \frac{\partial^2}{\partial \kappa^2} C(\kappa,u,K,T,\theta)\Big|_{\kappa=k}. \end{array}$$

The fact that weights do not depend on t and S guarantee that this relation can be thought as a hedging portfolio with no rebalancing until the expiring time u.

The last section of the first part of the article is focused on an approximation of the previous formula by the Gauss-Hermite quadrature rule. For any N, the Gauss-Hermite quadrature rule gives N weights $(w_i)_{i=1\ldots N}$ and N nodes $(x_i)_{i=1\ldots N}$ such that for any f which is $2N$-times differentiable, there exists $\xi \in (-\infty, \infty)$ such that :

$$\int_{-\infty}^{+\infty} f(x) e^{-x^2} dx = \sum_{i=1}^{N} w_i f(x_i) + \frac{N! \sqrt{\pi}}{2^N} \frac{f^{(2N)}(\xi)}{(2N)!}.$$

Then they suggest a mapping from $\mathbb{R}$ onto $\mathbb{R}_+$ to link the nodes x_i with some strikes K_i. Basically, the mapping is $K(x) = K e^{ax+b}$ where a and b depends on model parameters, the choice of u and the maturity.

1.2 Simulation analysis based on popular models

In this section, the authors compare their approach to standard dynamic delta hedging in the case of the Black Scholes model and the Merton model. They are interested in the following problem : the writer of a call option with maturity one year is going to hold this short position for a month and then he will close the position. They show that, in the case of jumps, the hedging error of the static porfolio with merely 3 options is significantly smaller than the one of the dynamic hedging portfolio.

2 What is implemented in Premia

In Premia, we have implemented the strike-nodes and weights for a portfolio of 5 call options with maturity u approximating the price of a call option with maturity T and strike K in the case of the Merton model.

More precisely, for u and T we give $(w_i)_{i=1\ldots5}$ and $(K_i)_{i=1\ldots5}$ such that given the model parameters θ :

$$\forall S \geq 0 \text{ and } t \in [0,u],\ C(S,t,K,T,\theta) \approx \sum_{i=1}^{5} w_i C(S,t,K_i,u,\theta).$$

References

[1] P. Carr and L. Wu. Static Hedging of Standard Options. *CRIF Working Paper seriesCRIF Working Paper series*, 22, 2004. 1