

cf_callmax

Let

- T = maturity date ($T > t$)
- K = strike price
- $x1$ = spot1 price
- $x2$ = spot2 price
- t = pricing date
- $\sigma1$ = volatility1
- $\sigma2$ = volatility2
- ρ = correlation
- r = interest rate
- $\delta1$ = dividend1 yields
- $\delta2$ = dividend2 yields
- $\theta = T - t$

Here, closed formulas due to Johnson and Stulz are presented [1],[2].

We set

- $d = \frac{\log \frac{x_1}{x_2} + \left(\delta_2 - \delta_1 + \frac{\sigma_2^2}{2} \right) \theta}{\sigma \sqrt{\theta}}$
- $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$
- $d_i = \frac{\log \left(\frac{x_i}{K} \right) + \left(r - \delta_i + \frac{\sigma_i^2}{2} \right) \theta}{\sigma_i \sqrt{\theta}}, \quad i = 1, 2$
- $\rho_1 = \frac{\sigma_1 - \rho\sigma_2}{\sigma}$
- $\rho_2 = \frac{\sigma_2 - \rho\sigma_1}{\sigma}$

and M as the cumulative bivariate normal distribution function:

$$M(a, b; \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^a \int_{-\infty}^b e^{-\frac{x^2-2\rho xy+y^2}{2(1-\rho^2)}} dx dy.$$

Call On Maximum Option

PAYOFF	$C_T = (\max(S_T^1, S_T^2) - K)_+$
PRICE	$C(t, x_1, x_2) = x_1 e^{-\delta_1 \theta} M(d_1, d; \rho_1) + x_2 e^{-\delta_2 \theta} M(d_2, -d + \sigma \sqrt{\theta}; \rho_2) - K e^{-r\theta} (1 - M(-d_1 + \sigma_1 \sqrt{\theta}, -d_2 + \sigma_2 \sqrt{\theta}; \rho))$
DELTA	$\frac{\partial C(t, x_1, x_2)}{\partial x_1} = e^{-\delta_1 \theta} M(d_1, d; \rho_1)$ $\frac{\partial C(t, x_1, x_2)}{\partial x_2} = e^{-\delta_2 \theta} M(d_2, -d + \sigma \sqrt{\theta}; \rho_2)$

References

- [1] H.JOHNSON. Options on the maximum ot the minimum of several assets.
J.Of Finance and Quantitative Analysis, 22:227–283, 1987. 1
- [2] R.STULZ. Options on the minimum or the maximum of two risky assets.
J. of Finance, 10:161–185, 1992. 1