

## tr\_kamradritchken\_bs

Input parameters:

- StepNumber  $N$
- StretchParameter  $\lambda$  (should be greater than 1)

Output parameters:

- Price
- Delta

This is taken from [1]. It is a 3-node tree which is the archetype of a trinomial tree. This is a flat tree with  $2N+1$  possible values of the underlying  $S_1$  throughout the option's life.

/\*Price, intrinsic value arrays\*/

/\*Up and Down factors\*/

Here  $u = e^{\lambda\sigma\sqrt{h}}$ ,  $d = e^{-\lambda\sigma\sqrt{h}}$ . The third node is  $m = 1$ .

/\*Discounted Probability\*/

These are computed by matching the two first moment conditions with a simplifying trick: the second moment condition is replaced by the equality of the second *moment* of the conditional random walk in the tree with the variance of the continuous limit logarithm of the Black-Scholes diffusion: the variances still match at order  $o(h)$  so that convergence follows from Kushner's theorem (cf [Convergence result for Tree methods in finance](#)) whereas the calculations are simpler. The computation is detailed [there](#). The stretch parameter  $\lambda$  is free with the following restrictions: it should be greater than 1 for the center-node probability to be positive and smaller

$$\text{than } \frac{\sigma}{\left| r - \text{diviv} - \frac{\sigma^2}{2} \right| \sqrt{h}}$$

/\*Intrinsic value initialisation and terminal values\*/

Since this is a flat tree we store the intrinsic values in an array as explained in [Routine tr\\_coxrossrubinstein\\_c](#).

/\*Backward Resolution\*/

Notice that the indexing of the price array  $P$  is relative to the lower of the underlying values at a fixed time whereas the intrinsic value array indexing  $iv$  is absolute. This accounts for the shift  $j$  in the index in

$$P[j] = \text{MAX}(iv[j+i], P[j])$$

/\*Delta\*/

We keep the formula of the CRR delta. Here it is no longer a perfect-hedging delta in the discrete-time scheme since this is an incomplete market. The convergence can be proved in the same manner as for the CRR delta (cf [there](#)). There maybe other more clever choices using the center node.

/\*First Time Step\*/

/\*Price\*/

/\*Memory desallocation\*/

## References

- [1] B.KAMRAD P.RITCHKEN. Multinomial approximating models for options with k state variables. *Management Science*, 37:1640–1652, 1991.

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