

## mc\_fixedasian\_privault

Input parameters:

- Number of iterations  $N$
- Generator\_Type
- Confidence Value
- Delta Method

Output parameters:

- Price  $P$
- Error Price  $\sigma_P$
- Delta  $\delta$
- Error delta  $\sigma_\delta$
- Price Confidence Interval:  $IC_P = [\text{Inf Price}, \text{Sup Price}]$
- Delta Confidence Interval:  $IC_\delta = [\text{Inf Delta}, \text{Sup Delta}]$

See [DocPrivault](#)

### Description:

Computation of the price and delta of a standard Asian option in a market with jumps.

We consider the following market model:

$$dS_t = rS_t dt + \sigma S_{t-} (\beta_{N_t-} dN_t - \nu dt),$$

where  $(N_t)_{t \in \mathbb{R}_+}$  is a Poisson process with constant intensity  $\lambda$ ,  $(\beta_k)_{k \in \mathbb{N}}$  is a sequence of random variables independent of  $(N_t)_{t \in \mathbb{R}_+}$ , and  $r$  represents the interest rate. For example, if  $(\beta_k)_{k \in \mathbb{N}}$  is a sequence of independent identically distributed random variables with:  $P(\beta_k = b_i) = p_i$ ,  $i = 1, \dots, d$ ,  $k \in \mathbb{N}$ , we have

$$\beta_{N_t-} dN_t = b_1 dN_t^1 + \dots + b_d dN_t^d$$

where  $N^1, \dots, N^d$  are independent Poisson processes with intensities

$$(\lambda_i)_{i=1, \dots, d} = (p_i \lambda)_{i=1, \dots, d},$$

and  $\nu = \lambda \sum_{i=1}^d b_i p_i$ . The Delta of an Asian option is given by

$$\Delta = \frac{\partial C}{\partial x},$$

where

$$C(x) = E \left[ f \left( \int_0^T S_u^x du \right) \right].$$

The derivative can be computed as

$$\frac{\partial}{\partial \zeta} E [f(F^\zeta)] = E \left[ \frac{\partial}{\partial \zeta} f(F^\zeta) \right] = E \left[ \frac{\partial}{\partial \zeta} F^\zeta f'(F^\zeta) \right],$$

however this formula requires a regularity property on  $f$ . Given a suitable function  $w$  and a Poisson functional  $F = f(T_1, \dots, T_n)$ , let  $D_w F$  be the gradient of  $F$  defined as

$$D_w F = - \sum_{k=1}^{k=n} w_{T_k} \partial_k f(T_1, \dots, T_n).$$

The adjoint of  $D$  satisfies the integration by parts formula:

$$E [\delta(u) F] = E [D_u F].$$

As a consequence, a first derivative such as Delta can be computed as follows:

$$\frac{\partial}{\partial \zeta} E [f(F^\zeta)] = E \left[ f(F^\zeta) \delta \left( w \frac{\partial_\zeta F^\zeta}{D_w F^\zeta} \right) \right], \quad (1)$$

with a weight given by

$$\delta \left( w \frac{\partial_\zeta F^\zeta}{D_w F^\zeta} \right) = \frac{\partial_\zeta F^\zeta}{D_w F^\zeta} \int_0^T \dot{w}_t dN_t - \frac{D_w \partial_\zeta F^\zeta}{D_w F^\zeta} + \frac{\partial_\zeta F^\zeta}{(D_w F^\zeta)^2} D_w D_w F^\zeta.$$

In the linear case  $F^x = xF$  and  $\partial_x F^x = F$ , the above formula simplifies:

$$\delta \left( w \frac{\partial_x F^x}{D_w F^x} \right) = \frac{1}{x} \left( \frac{F}{D_w F} \int_0^T \dot{w}_t d\tilde{N}_t - 1 + \frac{F}{(D_w F)^2} D_w D_w F \right).$$

The solution of the equation

$$dS_t = \alpha S_t dt + \sigma S_{t-} \beta_{N_{t-}} dN_t, \quad S_0 = x,$$

driving  $(S_t)_{t \in \mathbb{R}_+}$ , with  $\alpha = r - \nu\sigma$ , can be written as

$$S_t = F(t, N_t),$$

where

$$F(t, k) = x e^{\int_0^t \alpha_s(N_s) ds} \prod_{i=0}^{i=k} (1 + \beta_{i-1} \sigma).$$

The expression for the weight of Delta becomes:

$$\frac{1}{x\sigma} \left( \frac{\int_0^T S_t dt \int_0^T \dot{w}_t d\tilde{N}_t}{\int_0^T w_t S_{t-} \beta_{N_{t-}} dN_t} - 1 - \frac{\int_0^T S_t dt \int_0^T w_t (\dot{w}_t + \alpha w_t) S_{t-} \beta_{N_{t-}} dN_t}{\left( \int_0^T w_t S_{t-} \beta_{N_{t-}} dN_t \right)^2} \right).$$

## Algorithm:

/\* Generation of Exponential Law. Interjump Times \*/

The interjump times are generated as exponential random variables.

/\* Renormalization of sigma \*/

The value of the parameter  $\sigma$  is divided by  $\sqrt{\nu}$  in order to be consistent with the Black-Scholes model when the intensity  $\nu$  becomes large.

/\* Value to construct the confidence interval \*/

/\* Initialization \*/

/\*MC sampling\*/

Initialization of the simulation: generator type, dimension, size  $N$  of the sample, ... .

/\* Test after initialization for the generator \*/

/\* Begin N iterations \*/

/\* Simulation of Poisson Jump Times \*/

We simulate Poisson jump times with intensity  $\nu$  up to time  $T$  and determine their number.

/\* Computation of Average and the Weight \*/

Two methods are used, first the classical finite differences and secondly the Malliavin method.

/\* Useful for computation of the weight \*/

We compute the terms needed for the calculation of weights.

/\* Average \*/ Calculation of the average of the payoff.

/\* Price \*/ The price of a standard Asian option is computed.

/\* Delta \*/

/\* Finite Differences \*/ In this method the computation of delta is performed with:

$$\text{Delta}_{FD} = C(s + \epsilon) - C(s - \epsilon) / (2 * \epsilon)$$

/\* Malliavin \*/ In this case we use the expression of the weight :

$$\frac{1}{x\sigma} \left( \frac{\int_0^T S_t dt \int_0^T \dot{w}_t d\tilde{N}_t}{\int_0^T w_t S_{t-} \beta_{N_{t-}} dN_t} - 1 - \frac{\int_0^T S_t dt \int_0^T w_t (\dot{w}_t + \alpha w_t) S_{t-} \beta_{N_{t-}} dN_t}{\left( \int_0^T w_t S_{t-} \beta_{N_{t-}} dN_t \right)^2} \right).$$

/\*Sum\*/

Computation of the sums  $\sum P_i$  and  $\sum \delta_i$  for the mean price and the mean delta where

$$P(i) = f \left( \frac{1}{T} \int_0^T S_t^x(i) dt, K \right) \quad \text{and} \quad \delta_i = f \left( \frac{1}{T} \int_0^T S_t^x(i) dt, K \right) W(i),$$

with

$$W(i) = \frac{1}{x\sigma} \left( \frac{\int_0^T S_t dt \int_0^T \dot{w}_t d\tilde{N}_t}{\int_0^T w_t S_{t-} \beta_{N_{t-}} dN_t} - 1 - \frac{\int_0^T S_t dt \int_0^T w_t (\dot{w}_t + \alpha w_t) S_{t-} \beta_{N_{t-}} dN_t}{\left( \int_0^T w_t S_{t-} \beta_{N_{t-}} dN_t \right)^2} \right).$$

/\*Sum of squares\*/ Computation of the sums  $\sum P_i^2$  and  $\sum \delta_i^2$  necessary for the variance price and the variance delta estimations. (finally only used for MC estimation)

/\* End N iterations \*/

/\*Price\*/ The price estimator is:

$$P = \frac{1}{N} \exp(-rt) \sum_{i=1}^N P(i)$$

The error estimator is  $\sigma_P$  with :

$$\sigma_P^2 = \frac{1}{N-1} \left( \frac{1}{N} \exp(-2rt) \sum_{i=1}^N P(i)^2 - P^2 \right).$$

/\*Delta\*/ The estimator of  $\delta$  is:

$$\tilde{\delta} = \frac{1}{N} \exp(-rt) \sum_{i=1}^N f \left( \frac{1}{T} \int_0^T S_t^x(i) dt, K \right) W(i),$$

with

$$W(i) = \frac{1}{x\sigma} \left( \frac{\int_0^T S_t dt \int_0^T \dot{w}_t d\tilde{N}_t}{\int_0^T w_t S_{t-} \beta_{N_{t-}} dN_t} - 1 - \frac{\int_0^T S_t dt \int_0^T w_t (\dot{w}_t + \alpha w_t) S_{t-} \beta_{N_{t-}} dN_t}{\left( \int_0^T w_t S_{t-} \beta_{N_{t-}} dN_t \right)^2} \right).$$

The error estimator is  $\sigma_\delta$  with:

$$\sigma_\delta^2 = \frac{1}{N-1} \left( \frac{1}{N} \exp(-2rt) \sum_{i=1}^N \delta(i)^2 - \delta^2 \right)$$

/\* Price Confidence Interval \*/ The confidence interval is given as:

$$IC_P = [P - z_\alpha \sigma_P; P + z_\alpha \sigma_P]$$

with  $z_\alpha$  computed from the confidence value.

/\* Delta Confidence Interval \*/ The confidence interval is given as:

$$IC_\delta = [\delta - z_\alpha \sigma_\delta; \delta + z_\alpha \sigma_\delta]$$

with  $z_\alpha$  computed from the confidence value.

Possible improvement: in this program the function `jump_size` yields a constant vector equal to 1 everywhere, but it can be implemented as a random variable.