Heston-Hull-White model with ADI finite difference schemes

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1 Heston-Hull-White model and ADI schemes

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We consider the following model for the stock price:

$$\left\{ \begin{array}{rcl} dS_t & = & R_t S_t + \sqrt{V_t} S_t dW_t^S, \\ dV_t & = & \alpha_v(\beta_v - V_t)dt + \sigma_v \sqrt{V_t} dW_t^V, \\ dR_t & = & \alpha_r(b(t) - V_t)dt + \sigma_r dW_t^R, \end{array} \right. \tag{1}$$

where $b(t)$ is a given deterministic function of time called the mean reversion level. It can be chosen flat or you can use a given zero-coupon bond curve as input. Moreover there is correlation factors between the three brownian motion given by ρ_{sv}, ρ_{sr} and ρ_{vr}.

Moreover there is dividend (divid) and possibility to consider an european or an american option (call or put), given initial values $S = S_0$, $V = V_0$ and $R = R_0$, whose default values are given respectively by 100, 0.01 and 0.04. The maturity is one year and the strike value is 100.

Under these assumptions, we solve the following PDE

$$\begin{array}{rcl} \dfrac{\partial u}{\partial t} & = & \dfrac{1}{2} s^2 v \dfrac{\partial^2 u}{\partial s^2} + \dfrac{1}{2}\sigma_v^2 v \dfrac{\partial^2 u}{\partial v^2} + \dfrac{1}{2}\sigma_r^2 \dfrac{\partial^2 u}{\partial r^2} \\ & & + \rho_{sv}\sigma_v s v \dfrac{\partial^2 u}{\partial s \partial v} + \rho_{sr}\sigma_r s\sqrt{v} \dfrac{\partial^2 u}{\partial s \partial r} + \rho_{vr}\sigma_v\sigma_r\sqrt{v} \dfrac{\partial^2 u}{\partial v \partial r} \\ & & + rs\dfrac{\partial u}{\partial s} + \alpha_v(\beta_v - v)\dfrac{\partial u}{\partial v} + \alpha_r(b(T-t) - r)\dfrac{\partial u}{\partial r} - ru, \end{array} \tag{2}$$

with the following boundary conditions

$$\begin{aligned}
u(s,v,r,t) &= 0 && \text{whenever } s = 0, \\
\frac{\partial u}{\partial s}(s,v,r,t) &= \exp(-divid\ t) && \text{whenever } s = S_{\max}, \\
u(s,v,r,t) &= s\exp(-divid\ t) && \text{whenever } v = V_{\max}, \\
\frac{\partial u}{\partial r}(s,v,r,t) &= 0 && \text{whenever } r = \pm R_{\max},
\end{aligned}$$

for the call option, and

$$\begin{aligned}
u(s,v,r,t) &= K\exp(-rt) && \text{whenever } s = 0, \\
\frac{\partial u}{\partial s}(s,v,r,t) &= 0 && \text{whenever } s = S_{\max}, \\
u(s,v,r,t) &= K\exp(-rt) && \text{whenever } v = V_{\max}, \\
\frac{\partial u}{\partial r}(s,v,r,t) &= 0 && \text{whenever } r = \pm R_{\max},
\end{aligned}$$

for the put option.

We refer to [1] where the complete method is described. We have used the same grids whose sizes are given respectively for time, S-space, V-space and R-space by N_t, N_s, N_v and N_r. The default values are 100, 50, 10 and 10. This choice ensures very good estimations for the prices of call or put options in a large variety of parameters in approximately less than 10 seconds.

The Douglas scheme described in [1] has been implemented, but the methods for all the others schemes are potentially already in the code.

2 Implementation

The implementation of the ADI finite difference scheme is very complex, especially for this three dimensional model.

First we have implemented the functions to read the initial curve for the mean reversion level of the rate. The function $b(\cdot)$ is given by `func_zero_coupon`.

Next we have implemented all the methods to build the grids. They have explicit names (`grid_generation_HHW_spot`,...).

Then, we have implemented very useful functions to avoid creating large band matrix. Technically speaking, a point in the grid is linked (by the finite difference scheme) to 28 other points in the grid. We speak about a 29-stencil scheme (1 point + 28 other points). So each matrix are four-dimensional matrix (three dimensions + the 29-stencil scheme). We have also implemented the functions for the boundary conditions and the building of all the matrix.

Finally, we have implemented all the methods to solve the linear systems involved in the ADI schemes. These methods have explicit names
(`computation_explicit_syslin_spot_matrix`,
`computation_implicit_syslin_var_matrix`,...).

The function `ADI_HHW` is only the temporal loop with the ADI scheme. It makes all the allocations and de-allocations of all grids, three dimensional vectors and four dimensional matrix.

References

[1] Tinne Haentjens and Karel J. in't Hout. ADI finite difference schemes for the Heston-Hull-White PDE. J. Comp. Finan. 16, 83-110 (2012). 2