

tr_dermankani

Input parameters:

- StepNumber N

Output parameters:

- Price
- Delta

This algorithm is taken from [1]. It gives a solution to the problem of pricing Barrier Options with a Tree method. The issue, as it is discussed [there](#), is to provide the tree algorithm with a better value of the barrier than the crude approximation with the nearest level in the tree: indeed the spacing of the tree grid in the space direction is of order \sqrt{h} so that the naive barrier-version of the CRR algorithm, for instance, can not yield a convergence order better than \sqrt{h} . Numerical tests ([1]) show that this is definitely too crude. The idea of the algorithm as discussed [there](#) is to take as a price value for the node just above the barrier (let us denote this level $S_0 d^{eta0}$) the linear interpolation between the price in case the barrier is exactly at the level of the node (“modified barrier” $mod = S_0 d^{eta0}$ -the price of the option is then given by the *rebate* by definition of the contract) and the price in case the barrier is at the level of the node below (“effective barrier” $eff = S_0 d^{eta0+1}$):

$$C = \frac{(L - eff)}{(mod - eff)} C(L = mod) + \frac{(mod - L)}{(mod - eff)} C(L = eff)$$

This is a CRR tree with a modification of the backward expectation formula at the critical mesh breaching the barrier.

/*Price, intrinsic value arrays*/

This is a flat tree so we store the intrinsic values in an array *iv* to avoid recomputation at each node.

/*Up and Down factors*/
Exactly those of CRR.

/*Risk-Neutral Probability*/
Same remark.

/*Number of down moves just before breaching the barrier*/
Here we compute the value of the integer *eta0* above. Note that it may be 0, ie the barrier is breached at the first mesh of the tree. This requires a particular handling of the calculation of the delta.

/*Weights for the linear interpolation at the critical node*/
/*Node above the barrier*/
Here we compute the above weights $pl = \frac{(L-eff)}{(mod-eff)}$

/*Intrinsic value initialization*/
We start the indexing from above. The index of the critical node is **N+eta0** . There is no need to compute the intrinsic values for the nodes below.

/*Terminal Values*/
npoints is the index of the critical node among the values of the underlying at maturity. We store the value of the option at the effective barrier *eff* in **P[npoints+1]** .

/*Backward Resolution*/
We begin with the case **eta0>0** where the first mesh does not breach the barrier. In this case there are 3 stages: first the barrier is active, then (backward) the barrier is strictly below the lower node of the tree and the algorithm is exactly that of a CRR tree.

/*First part-the barrier is active*/
Every two steps in time the lower mesh of the tree (with root node index **npoints** breaches the barrier (**odd=1**) or not (**odd=0**). In the breaching case, the computation

```
P[j]=pu*P[j]+ pd*P[j+1];
if (am)
P[j]=MAX(iv[i+2*j],P[j]);
```

for $j = npoints$ computes the value $C(L = eff)$. Notice that we use here the value $P[npoints+1]$ computed before. Then we implement the interpolation in : $P[npoints] = pl * rebate + one_{minus_{pl}} * P[npoints]$ Next if necessary we store the value of the *rebate* in $P[npoints+1]$

/*Second part-the barrier is strictly below the tree*/
 From now on this is the standard CRR algorithm (cf.
[Routine tr_coxrossrubinstein_c](#)).

/*Delta*/

/*First time step*/

/*eta0=0, the first mesh breaches the barrier*/
 /*The barrier is always active*/

Then in order to compute the delta we stop the above First part algorithm
 at the first time step. We store the price value at the node $(2h, S_0)$ in
 flatprice.

/*For the delta*/

Here we store the value at the node $(h, u * S_0)$ in upprice.

/*First Time Step*/

Since the first mesh breaches the barrier we use the interpolation formula.

/*Price*/

/*Delta*/

The delta is computed as a finite-difference approximation, at time h ,
 between the value of the price at node $(h, u * S_0)$ and at node (h, S_0) . The
 value of the price at this last node is the interpolation between the price at
 the nodes $(0, S_0)$ and the nodes $(2h, S_0)$.

/*Memory Desallocation*/

References

- [1] E.DERMAN I.KANI D.ERGENER I.BARDHAN. Enhanced numerical
 methods for options with barriers. *Financial Analyst Journal*, pages 65–
 74, Nov-Dec 95 1995. [1](#)