

## fd\_fmgh2d

### Input parameters:

- Number of grids  $l$
- TimeStepNumber  $M$
- Epsilon

### Output parameters:

- Price
- Delta1
- Delta2

We use a multigrid algorithm based on the "Howard algorithm" (policy iteration) [3] and the multigrid method [4]. We refer to Akian [1] for a detailed presentation.

**/\*StepNumber  $N$ \*/**  
 $N = nn(l) + 1$  where  $nn(l)$  calculates the number of points in the grid of level  $l$ .

**/\*Memory Allocation\*/**

**/\*Covariance Matrix\*/**

**/\*Space localisation\*/**  
 Define the integration domain  $D = [-limit, limit]^2$  using probabilistic estimation.

**/\*Space Step\*/**

Define the space step  $h = \frac{2 * limit}{N}$ .

**/\*Time Step\*/**

Define the time step  $k = \frac{T}{M}$ .

**/\*Terminal Values\*/**

Put the value of the payoff into a vector  $P$

**/\*Homegenous Dirichlet Conditions\*/**

**/\*Factor of scheme\*/**

Initialize the matrix  $M^h$  issued from the discretization of the operator  $A$  in the case of Dirichlet Boundary conditions.

**/\*Finite difference Cycle\*/**

At any time step, we have to solve the linear complementarity problem.

**/\*Init pp and R\*/**

**/\*Howard cycle\*/**

We solve the linear complementarity problem using the Howard algorithm, which consists in constructing a convergent sequence  $u^p$  whose limit is  $u$ .

Let  $\epsilon > 0$  be given.

**Step 1** Let  $u^k$  be given, we compute  $(i, j) \rightarrow pp^k[i][j] = \operatorname{argmin}(M^{pp}u^k(i, j) - f^{pp}[i][j])$  where  $pp = 0$  or  $1$  (the domain is divided into 2 regions: the continuation region and the exercise region),  $M^0$  is the matrix  $M^h$  issued from the discretization of the operator  $A$ ,  $M^1 = Id$ ,  $f^0 = R$ ,  $f^1 = Obst$ .

**Step 1** We solve the linear system  $M^{pp^k}u = G^{pp^k}$  by the multigrid method. It gives  $u^{k+1}$ .

The stopping criteria is

$$\|u^{k+1} - u^k\|_{\infty} < \epsilon. \quad (1)$$

**/\*Price\*/**

**/\*Delta\*/**

**/\*Memory Desallocation\*/**

## References

- [1] Akian, M.: Méthodes multigrilles en contrôle stochastique. Thèse de doctorat de l'université Paris 9 Dauphine. (1990) [1](#)
- [2] Hackbusch, W.: Multi-grid methods and applications. Springer-Verlag. (1985)
- [3] Howard, R.A.: Dynamic Programming and Markov Process. (MIT Press. 1960) [1](#)
- [4] Mc Cormick, S.F.: Multigrid methods. SIAM frontiers in applied mathematics. 5 (1987). [1](#)