Documentation of `mc_bayer_roughbergomi`

Premia 22

Compute the European call option price of an asset in the rough Bergomi model.

Usage

The function takes 9 parameters, which are explained in Table 1. Note that the risk free interest rate is set to $r = 0$ here.

Parameter	Interpretation	Range
S_0	Spot price of underlying	$\mathbb{R}_{>0}$
η	Parameter of the stochastic volatility, "vol-of-vol"	$\mathbb{R}_{>0}$
H	Hurst index of the fractional Brownian motion	$]0, 1/2[$
ρ	Correlation between Bm driving spot and volatility	$]-1, 1[$
ξ	Spot variance	$\mathbb{R}_{>0}$
K	Strike price of the option	$\mathbb{R}_{>0}$
T	Maturity of the option (years)	$\mathbb{R}_{>0}$
M	# of samples for Monte Carlo simultion	$\mathbb{N}$
N	# of grid points for the time discretization	$\mathbb{N}$

Table 1: Parameters of `mc_bayer_roughbergomi`

Background

`mc_bayer_roughbergomi` computes the price of a European call option in the rough Bergomi model by Bayer, Friz and Gatheral [BFG] using Monte Carlo simulation based on the hybrid scheme of Bennedsen, Lunde and Pakkanen [BLP].

The rough Bergomi model is described by the dynamics

$$dS_t = \sqrt{v_t} S_t dZ_t,$$
$$v_t = \xi_0(t) \exp\left(\eta \widetilde{W}_t - \frac{1}{2}\eta^2 t^{2H}\right),$$

where W, Z denote two *correlated* standard Brownian motions–with correlation ρ–and $\eta > 0$ is interpreted as a volatility of volatility parameter. More interestingly, for a Hurst parameter $0 < H < 1$ we have

$$\widetilde{W}_t = \int_0^t K(t,s) dW_s, \quad K(t,s) = \sqrt{2H}(t-s)^{H-1/2}.$$

This defines a variant of the fractional Brownian motion, sometimes called "Riemann-Liouville fractional Brownian motion", which is not the standard fBm. Finally, ξ_0 is the forward variance curve, which is assumed to be constant in this implementation.

Notice that samples from $\widetilde{W}$ cannot be obtained by standard algorithms for fractional Brownian motion, as $\widetilde{W}$ does not have stationary increments.

The *hybrid scheme* [BLP] used here provides values of $\widetilde{W}$ sampled along a grid of size N in computational time of order $N \log N$ (instead of order N^2 for a Cholesky scheme), but the distribution of the samples is not exact. However, [BLP] report that the accuracy is adequate for practical purposes for the rough Bergomi model.

References

[BFG] Ch. Bayer, P. Friz, J. Gatheral: Pricing under rough volatility, *Quantitative Finance* 16(6), 887-904, 2016.

[BLP] M. Bennedsen, A. Lunde, M. S. Pakkanen: Hybrid scheme for Brownian semistationary processes, *Finance and Stochastics* 21(4), 931-965, 2017.

1