

cf_call_merhes

The evolution process of the Heston model, for the stochastic volatility, and Merton model, for the jumps, is:

$$\begin{cases} \frac{dS_t}{S_t} &= (r - d)dt + \sqrt{V_t}dW_t^1 + (e^J - 1)dN_t \\ dV_t &= \kappa(\theta - V_t)dt + \sigma_v\sqrt{V_t}dW_t^2 \\ S(t=0) &= S_0 \\ V(t=0) &= V_0 \end{cases}$$

where $d < W^1, W^2 >_t = \rho dt$ and $J \sim N(m, v)$.

For European options, two pricing formula are giving based on the Fourier transform method [1]. In this document, we use the following notation: $\varphi = +1$ for a call and $\varphi = -1$ for a put; $\tau = T - t$; $x_t = \ln(S_t)$ and $X = \ln(S_t/K) + (r - d)\tau$.

1 The characteristic formula

The price $F(x_t, t)$ is given by:

$$F(x_t, t) = \frac{1 + \varphi}{2} e^{x_t - (T-t)d} + \frac{1 - \varphi}{2} e^{1 - (T-t)r} K - e^{-(T-t)r} f(x, V, \lambda, t)$$

where

$$f(x, V, \lambda, \tau) = \frac{K}{\pi} \int_0^\infty \Re \left[\frac{Q(k, x, V, \lambda, \tau)}{k^2 + 1/4} \right] dk$$

with

$$Q(k, x, V, \lambda, \tau) = e^{(-ik + 1/2)X + A(k, \tau) + B(k, \tau)V_0 + C(k, \tau) + D(k, \tau)\lambda}$$

The hedge δ is given by:

$$\delta = \frac{1 + \varphi}{2} e^{-(T-t)d} - e^{-(T-t)r} \frac{K}{\pi} \int_0^\infty \Re \left[\frac{(1/2 - ik)}{S_t} \cdot \frac{Q(k, x, V, \lambda, \tau)}{k^2 + 1/4} \right] dk$$

The coefficients $A(k, \tau)$, $B(k, \tau)$, $C(k, \tau)$ and $D(k, \tau)$ are specified as follows:

1. Volatility:

- Constant volatility :

$$A(k, \tau) = 0, B(k, \tau) = -1/2(k^2 + 1/4)\tau.$$

- Stochastic volatility (Heston) :

$$A(k, \tau) = -\frac{\kappa\theta}{\sigma_v^2} \left[\psi_+ \tau + 2 \ln \left(\frac{\psi_- + \psi_+ e^{-\tau\zeta}}{2\zeta} \right) \right]$$

$$B(k, \tau) = -(k^2 + 1/4) \frac{1 - e^{-\tau\zeta}}{\psi_- + \psi_+ e^{-\tau\zeta}}$$

where $\psi_{\pm} = \mp(u + ik\rho\sigma_v) + \zeta$, $\zeta = \sqrt{k^2\sigma_v^2(1 - \rho^2) + 2ik\rho\sigma_v u + u^2 + \sigma_v^2/4}$
and $u = \kappa - \rho\sigma_v/2$.

2. Jumps :

- Merton model : constant jump rate intensity and log-normal jump size distribution

$$C(k, \tau) = 0, D(k, \tau) = \tau\Lambda(k) \text{ where}$$

$$\Lambda(k) = e^{-ik(m+v^2/2) - (k^2-1/4)v^2/2 + 1/2m} - 1 - (-ik + 1/2)(e^{m+v^2/2} - 1).$$

2 The Black-Scholes-style fomula

The price $F(x_t, t)$ is given by

$$F(x_t, t) = \varphi \left(e^{-d(T-t)} S_t P_1(\varphi) - e^{-r(T-t)} K P_2(\varphi) \right)$$

and the hedge δ by

$$\delta = \varphi e^{-d(T-t)} P_1(\varphi)$$

where $P_j(\varphi) = \frac{1-\varphi}{2} + \varphi \Pi_j$ for $j \in \{1, 2\}$ with

$$\Pi_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \Re \left[\frac{\phi_j(k)}{ik} \right] dk$$

where the characteristic functions ϕ_j , for $j \in \{1, 2\}$, are given by:

$$\phi_j(k) = e^{ikX + A(k, \tau) + B(k, \tau)V_0 + C(k, \tau) + D(k, \tau)\lambda}$$

Using the notations :

$$\begin{cases} u = +1, & I = 1, & b = \kappa - \rho\sigma_v & \text{if } j = 1 \\ u = -1, & I = 0, & b = \kappa & \text{if } j = 2 \end{cases}$$

the coefficients $A(k, \tau)$, $B(k, \tau)$, $C(k, \tau)$ and $D(k, \tau)$ are given as follows:

1. Volatility:

- Constant volatility :
 $A(k, \tau) = 0, B(k, \tau) = -1/2(k^2 - uik)\tau.$
- Stochastic volatility (Heston) :

$$A(k, \tau) = -\frac{\kappa\theta}{\sigma_v^2} \left[\psi_+ \tau + 2 \ln \left(\frac{\psi_- + \psi_+ e^{-\tau\zeta}}{2\zeta} \right) \right]$$

$$B(k, \tau) = -(k^2 - uik) \frac{1 - e^{-\tau\zeta}}{\psi_- + \psi_+ e^{-\tau\zeta}}$$

$$\text{where } \psi_{\pm} = \mp(b - \rho\sigma_v ik) + \zeta \text{ and } \zeta = \sqrt{(b - \rho\sigma_v ik)^2 + \sigma_v^2(k^2 - iuk)}.$$

2. Jumps :

- Merton model : constant jump rate intensity and log-normal jump size distribution
 $C(k, \tau) = 0, D(k, \tau) = \tau\Lambda(k)$ where
 $\Lambda(k) = e^{(m+Iv^2)ik - v^2k^2/2 + I(m+v^2/2)} - 1 + (ik + I)(e^{m+v^2/2} - 1).$

3 Numerical integration

In order to compute the infinite integrals, needed in the pricing formulas, we use the approximation:

$$\int_0^\infty f(x)dx \simeq \sum_{j=0}^N \int_{jh}^{(j+1)h} f(x)dx.$$

The number N of the sub-integrals used is determined when the contribution of the last strip $[jh, (j+1)h]$ is smaller than a given tolerance ϵ . Each sub-integral $\int_{jh}^{(j+1)h} f(x)dx$ is computed using a Gaussian quadrature.

4 Implementation of the pricing routines

4.1 The `svj.c` file

This file contains the pricing routines, giving the price of an european call or put in the Merton/Heston/Merton+Heston models. Any of the two pricing formulas presented in Sections 1 and 2 can be used. *Opt* is the option type, *call* or *put*. *Model* is the model used, *merton* for the Merton model, *heston* for the Heston model and *hestmert* for the combined model Heston+Merton.

In each file, we set the option type and the model parameters, next, we call the `calc_price_svj` routine from `svj.c` file. The default pricing method used is the Black-Scholes like formula given in [2](#).

References

- [1] A.Sepp. Pricing european-style options under jump diffusion processes with stochastic volatility: Applications of fourier transform. *Proceedings of the 7th Tartu Conference on Multivariate Statistics*, 2004. [1](#)