EFFICIENT AND ACCURATE LOG-LÉVY APPROXIMATIONS TO LÉVY DRIVEN LIBOR MODELS *PREMIA DOCUMENTATION*

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Premia 22

1. Introduction

The LIBOR market model is very popular for pricing interest rate derivatives, but is known to have several pitfalls. In addition, if the model is driven by a jump process, then the complexity of the drift term is growing exponentially fast (as a function of the tenor length). In this work, we consider a Lévy-driven LIBOR model and aim at developing accurate and efficient log-Lévy approximations for the dynamics of the rates. The approximations are based on truncation of the drift term and Picard approximation of suitable processes. This document is based on the paper 10) which can be referred to for more details as well as other alternative approximations.

2. Lévy LIBOR framework

Let $0 = T_0 < T_1 < \cdots < T_N < T_{N+1} = T_*$ denote a discrete tenor structure where $\delta_i = T_{i+1} - T_i$, $i = 0, 1, \ldots, N$, are the so called day-count fractions. For this tenor structure we consider an arbitrage free system of zero coupon bond processes B_i, $i = 1, \ldots, N+1$, on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0 \le t \le T_*}, \mathbb{P}_*)$, where $\mathbb{P}_* := \mathbb{P}_{N+1}$ is a numeraire measure connected with the terminal bond B_{N+1}. From this bond system we may deduce a forward rate system, also called LIBOR rate system, defined by

$$L_i(t) := \frac{1}{\delta_i}\left(\frac{B_i(t)}{B_{i+1}(t)} - 1\right), \quad 0 \le t \le T_i,\ 1 \le i \le N. \tag{2.1}$$

L_i is the annualized effective forward rate contracted at date $t \le T_i$ for the period $[T_i, T_{i+1}]$. 7) derived a general representation for the LIBOR dynamics in a semimartingale framework. In this article we consider a Lévy LIBOR framework as constructed by 4); see also 5) and 2) for jump-diffusion settings.

Consider a standard Brownian motion W in $\mathbb{R}^m$, $m \le N$, a bounded deterministic nonnegative scalar function $\alpha(s)$, $s \in [0, T_*]$, and a random measure μ on $[0, T_*] \times \mathbb{R}^m$ with $\mathbb{P}_*$-compensator $F(s, \mathrm{d}x)\mathrm{d}s$, where μ and W

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are mutually independent. Let $H = (H(t))_{0\leq t\leq T_*}$ be a time-inhomogeneous Lévy process with canonical decomposition

$$H(t) = \int_0^t \sqrt{\alpha(s)}\mathrm{d}W(s) + \int_0^t \int_{\mathbb{R}^m} x(\mu(\mathrm{d}s,\mathrm{d}x) - F(s,\mathrm{d}x)\mathrm{d}s). \tag{2.2}$$

We denote by $\widetilde{\mu}$ the compensated random measure of the jumps of H, that is $\widetilde{\mu}(\mathrm{d}s,\mathrm{d}x) := \mu(\mathrm{d}s,\mathrm{d}x) - F(s,\mathrm{d}x)\mathrm{d}s$. In order to avoid truncation conventions we assume that F satisfies the (stronger than usual) integrability condition

$$\int_0^{T_*} \int_{\mathbb{R}^m} \left(\|x\| \wedge \|x\|^2\right) F(s,\mathrm{d}x)\mathrm{d}s < \infty.$$

We further assume that

$$\int_0^{T_*} \int_{\|x\|>1} \exp\left(u^\mathsf{T} x\right) F(s,\mathrm{d}x)\mathrm{d}s < \infty, \tag{2.3}$$

for all $\|u\| \leq (1+\varepsilon)\overline{M}$, with $\overline{M}, \varepsilon > 0$ constants. Thus, by construction, the process $(H(t))_{0\leq t\leq T_*}$ is a $\mathbb{P}_*$-martingale. The cumulant generating function of $H(t)$, $t \in [0,T_*]$, is provided by

$$\ln \mathbb{E}[\mathrm{e}^{u^\mathsf{T} H(t)}] = \kappa_t(u) = \frac{\alpha(t)}{2}\|u\|^2 + \int_{\mathbb{R}^m} (\mathrm{e}^{u^\mathsf{T} x} - 1 - u^\mathsf{T} x) F(t,\mathrm{d}x). \tag{2.4}$$

Along with the Lévy martingale (2.2) we introduce a set of bounded deterministic vector-valued functions $\lambda_i(s) \in \mathbb{R}^m$, $i = 1,\dots,N$, usually called *loading factors*. In order to avoid local redundances we assume that the matrix $[\lambda_1,\dots,\lambda_N](s)$ has full rank m for all $s \in [0,T_*]$. Moreover, we assume that $\|\lambda_i(s)\| \leq \overline{M}$, for all i, and $\|\sum_i \lambda_i(s)\| \leq \overline{M}$, for all $s \in [0,T_*]$.

The Lévy martingale and the set of loading factors then constitute an arbitrage free LIBOR system consistent with (2.1), whose dynamics under the terminal measure $\mathbb{P}_*$ are given by

$$L_i(t) = L_i(0) \exp\left(\int_0^t b_i(s)\mathrm{d}s + \int_0^t \lambda_i^\mathsf{T}(s)\mathrm{d}H(s) \right), \tag{2.5}$$

$i = 1,\dots,N$, where the drift terms in the exponent are given by

$$\begin{aligned} b_i = &-\frac{1}{2}\alpha\left|\lambda_i\right|^2 - \sum_{j=i+1}^N \frac{\delta_j L_{j-}}{1+\delta_j L_{j-}} \alpha \lambda_i^\mathsf{T} \lambda_j \\ &- \int_{\mathbb{R}^m} \left(\left(\mathrm{e}^{\lambda_i^\mathsf{T} x} - 1\right) \prod_{j=i+1}^N \left(1 + \frac{\delta_j L_{j-}\left(\mathrm{e}^{\lambda_j^\mathsf{T} x} - 1\right)}{1+\delta_j L_{j-}} \right) - \lambda_i^\mathsf{T} x \right) F(\cdot,\mathrm{d}x); \end{aligned} \tag{2.6}$$

for details see 4). For notational convenience, we set $L_{j-}(s) := L_j(s-)$ in (2.6), while the time variable is suppressed.

Due to the drift term (2.6), a straightforward Monte Carlo simulation of (2.5) would involve a numerical integration at each time step, since the random terms $\frac{\delta_j L_{j-}}{1+\delta_j L_{j-}}$ appear under the integral sign. In order to overcome this problem, we will re-express the drift in terms of random quotients multiplied with cumulants of the driving process. We have that

$$
\begin{aligned}
b_i = -\kappa(\lambda_i) &- \sum_{j=i+1}^{N} \frac{\delta_j L_{j-}}{1+\delta_j L_{j-}} \alpha \lambda_i^\top \lambda_j \\
&- \sum_{p=1}^{N-i} \sum_{i<j_1<\cdots<j_p\leq N} \frac{\delta_{j_1} L_{j_1-}}{1+\delta_{j_1} L_{j_1-}} \cdots \frac{\delta_{j_p} L_{j_p-}}{1+\delta_{j_p} L_{j_p-}} \\
&\times \sum_{q=1}^{p+1} (-1)^{p+q+1} \sum_{0\leq r_1<\cdots<r_q\leq p} \widehat{\kappa}(\lambda_{j_{r_1}} + \cdots + \lambda_{j_{r_1}});
\end{aligned}
\tag{2.7}
$$

the derivation is deferred to Appendix A of 10). Here $\widehat{\kappa}$ denotes the part of the cumulant κ stemming from the jumps of L, that is

$$
\widehat{\kappa}_s(u) = \int\limits_{\mathbb{R}^m} (\mathrm{e}^{u^\top x} - 1 - u^\top x) F(s, \mathrm{d}x). \tag{2.8}
$$

Therefore, we can now avoid the numerical integration when simulating LIBOR rates. However, another problem becomes apparent in this representation: the number of terms to be computed in (2.7) grows exponentially fast as a function of the number of LIBOR rates N, namely it has order $O(2^N)$.

3. Efficient and accurate log-Lévy approximations

The aim of this section is to derive efficient and accurate log-Lévy approximations for the dynamics of the LIBOR rates under the terminal measure. This is based on an appropriate approximation of the drift term, cf. (2.6), which has two pillars:

(1) expansion and truncation of the drift term,
(2) Picard approximation of suitably defined processes.

3.1. **Log-Lévy approximation schemes.** In the sequel, we are going to follow this recipe for deriving efficient and accurate log-Lévy approximations, and present the full details of the method. However, we will first truncate the drift terms at the second order

1. The first step is to expand and truncate the drift term at the second order, that is we will approximate b_i by b_i'', where

$$
\begin{aligned}
b_i'' = -\theta_i \; &- \sum_{i+1\leq j\leq N} \frac{\delta_j L_{j-}}{1+\delta_j L_{j-}} \eta_{ij} \\
&- \sum_{i+1\leq k<l\leq N} \frac{\delta_k L_{k-}}{1+\delta_k L_{k-}} \frac{\delta_l L_{l-}}{1+\delta_l L_{l-}} \zeta_{ikl},
\end{aligned}
\tag{3.1}
$$

where

$$
\theta_i = \kappa(\lambda_i), \qquad \eta_{ij} = \kappa(\lambda_i + \lambda_j) - \kappa(\lambda_i) - \kappa(\lambda_j) \tag{3.2}
$$

and

$$
\begin{aligned}
\zeta_{ikl} = \widehat{\kappa}(\lambda_i + \lambda_k + \lambda_l) &- \widehat{\kappa}(\lambda_i + \lambda_k) - \widehat{\kappa}(\lambda_i + \lambda_l) \\
&- \widehat{\kappa}(\lambda_k + \lambda_l) + \widehat{\kappa}(\lambda_i) + \widehat{\kappa}(\lambda_k) + \widehat{\kappa}(\lambda_l).
\end{aligned}
\tag{3.3}
$$

The number of terms to be calculated is thus reduced from $O(2^N)$ to $O(N^2)$, while the error induced is

$$b_i = b_i'' + O(N^2\delta^3\|L\|^3). \tag{3.4}$$

Therefore, the gain in computational time is significant, while the loss in accuracy is usually relatively small see the numerical analysis in 10). The above approximation is referred to in Premia as the *second order drift expansion.*

2. The second step is to approximate the random terms

$$Z_j(t) := \frac{\delta_j L_j(t)}{1+\delta_j L_j(t)} \quad \text{and} \quad Y_{kl}(t) := \frac{\delta_k L_k(t)}{1+\delta_k L_k(t)} \frac{\delta_l L_l(t)}{1+\delta_l L_l(t)} \tag{3.5}$$

in (3.1) by a time-inhomogeneous Lévy process. If we limit ourselves to a first order log-Lévy approximation we can disregard the random terms $Y_{kl}(t)$. Let us define,

$$f(x) = \frac{\delta_j \mathrm{e}^x}{1+\delta_j \mathrm{e}^x}$$

where

$$f'(x) = \frac{\delta_j \mathrm{e}^x}{(1+\delta_j \mathrm{e}^x)^2} \quad \text{and} \quad f''(x) = \frac{\delta_j \mathrm{e}^x(1-\delta_j \mathrm{e}^x)}{(1+\delta_j \mathrm{e}^x)^3}.$$

We obviously have that

$$Z_j(t) = f(G_j(t)) \tag{3.6}$$

The function f is C^2-differentiable, hence we can apply Itô's formula for semimartingales (cf. e.g. 6, Theorem I.4.57) to Z_j and derive (with time variable s suppressed or denoted by $\cdot$ in the integrands)

$$\begin{aligned} \mathrm{d}Z_j = \Bigg(\int_{\mathbb{R}^m} & \left(f(G_j + \lambda_j^\top x) - f(G_j) - f'(G_j)\,\lambda_j^\top x \right) F(\cdot, \mathrm{d}x) \\ & + f'(G_j)\, b_j'' + \frac{1}{2} f''(G_j)\, |\lambda_j|^2\, \alpha \Bigg) \mathrm{d}s + f'(G_j)\, \sqrt{\alpha} \lambda_j^\top \mathrm{d}W \\ + \int_{\mathbb{R}^m} & \left(f(G_{j-} + \lambda_j^\top x) - f(G_{j-}) \right) (\mu(\mathrm{d}s, \mathrm{d}x) - F(\cdot, \mathrm{d}x)\mathrm{d}s)\,. \end{aligned} \tag{3.7}$$

Hence, we have that

$$\begin{aligned} \mathrm{d}Z_j(s) \;\; = \;\; & A_j(s, L(s))\mathrm{d}s + B_j^\top(s, L_j(s))\mathrm{d}W(s) \\ & + \int_{\mathbb{R}^m} C_j(s, L_j(s), x)\, (\mu(\mathrm{d}s, \mathrm{d}x) - F(\cdot, \mathrm{d}x)\mathrm{d}s)\,, \end{aligned} \tag{3.8}$$

with obvious definitions of the *deterministic* functions A_j, B_j, and C_j. Due to the drift term b_j'', the function A_j depends on the whole LIBOR vector L rather than L_j only.

3. The next step is to approximate Z_j by a suitable Lévy processes. This approximation is based on a Picard iteration for the SDEs in (3.8). The initial value of the Picard iteration is

$$Z_j^{(0)} = Z_j(0) = \frac{\delta_j L_j(0)}{1+\delta_j L_j(0)}, \tag{3.9}$$

while the first order Picard iteration is provided by

$$
\begin{aligned}
Z_j^{(1)}(t) = Z_j(0) + \int_0^t A_j(s, L(0))\mathrm{d}s + \int_0^t B_j^{\mathsf{T}}(s, L_j(0))\mathrm{d}W(s) \\
+ \int_0^t \int_{\mathbb{R}^m} C_j(s, L_j(0), x)\,(\mu(\mathrm{d}s, \mathrm{d}x) - F(\cdot, \mathrm{d}x)\mathrm{d}s)\,. \qquad (3.10)
\end{aligned}
$$

We can easily deduce that $Z^{(1)}$ is a *time-inhomogeneous Lévy process*, since the coefficients $A_j(\cdot, L(0))$, $B_j(\cdot, L_j(0))$, and $C_j(\cdot, L_j(0), \cdot)$ in (3.10) are *deterministic*. Indeed, we have that

$$
\begin{aligned}
A_j(s, L(0)) = f'\left(G_j(0)\right) b_j^{(0)}(s) + \frac{1}{2} f''\left(G_j(0)\right) \left|\lambda_j\right|^2 (s)\alpha(s) \\
+ \int_{\mathbb{R}^m} \Big(f(G_j(0) + \lambda_j^{\mathsf{T}}(s)x) - f(G_j(0)) - f'\left(G_j(0)\right) \lambda_j^{\mathsf{T}}(s)x\Big)\, F(\cdot, \mathrm{d}x), \quad (3.11)
\end{aligned}
$$

where

$$
\begin{aligned}
b_j^{(0)}(s) := -\theta_i(s) \; - \sum_{i+1 \le j \le N} \frac{\delta_j L_{j-}(0)}{1 + \delta_j L_{j-}(0)} \eta_{ij}(s) \\
- \sum_{i+1 \le k < l \le N} \frac{\delta_k L_{k-}(0)}{1 + \delta_k L_{k-}(0)} \frac{\delta_l L_{l-}(0)}{1 + \delta_l L_{l-}(0)} \zeta_{ikl}(s),
\end{aligned}
$$

and

$$
\begin{aligned}
B_j(s, L_j(0)) &= f'\left(G_j(0)\right) \sqrt{\alpha(s)} \lambda_j(s), & (3.12) \\
C_j(s, L_j(0), x) &= f(G_j(0) + \lambda_j^{\mathsf{T}}(s)x) - f(G_j(0)). & (3.13)
\end{aligned}
$$

4. The fourth step is to apply the Lévy approximations of the random terms to (3.1). Let us denote by $\widehat{b}_i$ the resulting approximate drift term; we have that

$$
b_i'' \approx \widehat{b}_i := -\theta_i \; - \sum_{i+1 \le j \le N} \eta_{ij} Z_j^{(1)} \qquad (3.14)
$$

Keeping in mind that $\widehat{b}_i$ will be integrated over time, we define

$$
V_{ij}(s,t) = \int_s^t \eta_{ij}(r)\mathrm{d}r,
$$

which is seen to be a deterministic process of finite variation. Now, for fixed $t > 0$, we can apply integration by parts, which yields

$$
\begin{aligned}
\int_0^t \eta_{ij}(s) Z_j^{(1)}(s)\mathrm{d}s \overset{(3.10)}{=} V_{ij}(0,t)Z_j(0) &+ \int_0^t V_{ij}(s,t)A_j(s,L(0))\mathrm{d}s \\
&+ \int_0^t V_{ij}(s,t)B_j^\top(s,L_j(0))\mathrm{d}W(s) \qquad (3.15)\\
&+ \int_0^t V_{ij}(s,t)\int_{\mathbb{R}^m} C_j(s,L_j(0),x)\widetilde{\mu}(\mathrm{d}s,\mathrm{d}x).
\end{aligned}
$$

5. Finally, collecting all the pieces together we can derive a Lévy approximation for the log-LIBOR rates. The *approximate* log-LIBOR is denoted by $\widehat{G}_i$ and has the following dynamics

$$
\widehat{G}_i(t) = G_i(0) + \int_0^t \widehat{b}_i(s)\mathrm{d}s + \int_0^t \lambda_i^\top(s)\mathrm{d}H(s), \tag{3.16}
$$

which using (3.14) and (3.15) leads to

$$
\begin{aligned}
\widehat{G}_i(t) = \widehat{G}_i(0,t) &- \int_0^t \Bigg[\theta_i(s) + \sum_{i+1\le j\le N} V_{ij}(s,t)A_j(s,L(0))\Bigg]\mathrm{d}s \\
&+ \int_0^t \Bigg[\sqrt{\alpha(s)}\lambda_i^\top(s) - \sum_{i+1\le j\le N} V_{ij}(s,t)B_j^\top(s,L_j(0))\Bigg]\mathrm{d}W(s) \\
&+ \int_0^t\int_{\mathbb{R}^m} \Bigg[\lambda_i^\top(s)x - \sum_{i+1\le j\le N} V_{ij}(s,t)C_j(s,L_j(0),x)\Bigg]\widetilde{\mu}(\mathrm{d}s,\mathrm{d}x),
\end{aligned} \tag{3.17}
$$

with

$$
\widehat{G}_i(0,t) := G_i(0) - \sum_{i+1\le j\le N} V_{ij}(0,t)Z_j(0).
$$

Let us abbreviate (3.17) by

$$
\widehat{G}_i(t) = \widehat{G}_i(0,t) + \int_0^t \mathrm{H}_i(t,s)\mathrm{d}s + \int_0^t \Theta_i^\top(t,s)\mathrm{d}W(s) + \int_0^t \mathrm{I}_i(t,s,x)\widetilde{\mu}(\mathrm{d}s,\mathrm{d}x)
$$

Obviously, the above approximation is a time-inhomogeneous Lévy process whose characteristic function may be expressed by the Lévy–Khintchine formula in terms of H_i, Θ_i and I_i in a straightforward manner.

Remark 3.1. We will call the approximation in (3.17) the *first order log-Lévy approximation* of the LIBOR rate. The approximation can be further refined by including the second order terms (i.e. those depending on L_k and L_l) in (3.1). These terms can be approximated in a manner analogous to the Z_j's. See again 10) for more details.

4. Example: Pricing of Swaptions

The implementation considers the following simple example. We assume a a flat and constant volatility structure i.e. constant $\lambda_i's$. Similarly, zero coupon rates are generated from a flat term structure of LIBOR interest rates with equidistant tenor points. Furthermore we set $\alpha = 0$, thus limiting ourselves to the case where H is a pure jump Lévy process. In particular we choose H to be a CGMY process (cf. 3 and 9). The CGMY process has cumulant generating function defined for all $u \in \mathbb{C}$ with $|\Re u| \leq \min(G, M)$,

$$\begin{aligned}\kappa_{\text{CGMY}}(u) = \Gamma(-Y)G^Y &\left\{\left(1 - \frac{u}{G}\right)^Y - 1 + \frac{uY}{G}\right\} \\ &+ \Gamma(-Y)M^Y \left\{\left(1 + \frac{u}{M}\right)^Y - 1 - \frac{uY}{M}\right\}.\end{aligned} \tag{4.1}$$

The necessary conditions are then satisfied for term structures with volatility structures that satisfy $\sum_{i=1}^{N} |\lambda_i| \leq \min(G, M)$. Exact simulation of the increments can be performed without approximation using the approach in 11). This approach can be used when simulating with or without drift expansions, but cannot be employed in the case of the log-Lévy approximation in (3.17) where jump sizes are transformed in a non-linear fashion. Instead we employ an approximation where we replace jumps smaller than ϵ with their expectation which is zero since the jumps are compensated. This means that jumps bigger than ϵ follow a compound Poisson process which can be easily simulated using the so-called Rosinski rejection method (see 12 and 1, p. 338). We set the truncation point sufficiently low, at $\epsilon = 10^{-3}$, thus making the variance of the truncated term small enough to safely disregard. To be consistent, we employ this procedure everywhere we simulate from the CGMY process.

The payoffs we price are payer and receiver swaptions respectively. Following 8, pp. 78), we have that the price of a receiver swaption with strike rate K, where the underlying swap starts at time T_i and matures at T_m ($i < m \leq N$) is given by

$$\mathbb{S}_0 = B(0, T_*)\,\mathbb{E}_{\mathbb{P}_*}\left[\left(-\sum_{k=i}^{m}\left(c_k \prod_{l=k}^{N}(1 + \delta_i L_l(T_i))\right)\right)^+\right], \tag{4.2}$$

where

$$c_k = \begin{cases} -1, & k = i, \\ \delta_k K, & i + 1 \leq k \leq m - 1, \\ 1 + \delta_k K, & k = m. \end{cases} \tag{4.3}$$

Analogously, payer swaptions can be priced by merely replacing the minus inside the expecation in (4.2).

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