fd_howard

Input parameters:

- SpaceStepNumber N
- TimeStepNumber M
- Theta $\frac{1}{2} \leq \theta \leq 1$
- Epsilon

Output parameters:

- Price
- Delta

The algorithm of Howard has been introduced by Howard in [1].

/*Memory Allocation*/

/*Time Step*/
Define the time step $k = \dfrac{T}{M}$.

/*Space localisation*/
Define the integration domain $D = [-l, l]$ using the probabilistic estimate there.

/*Space Step*/
Define the space step $h = \dfrac{2l}{N}$.

/*Peclet Condition*/
If $|r - \delta| / \sigma^2$ is not small, then a more stable finite difference approximation is used. cf there.

/*Lhs factor of theta scheme*/
Initialize the matrix M^h issued from the discretization of the operator A in the case of Dirichlet Boundary conditions. cf there.

/*Rhs factor of theta scheme*/
Initialize the matrix N issued from the θ-scheme method in the cases of Dirichlet Boundary conditions. there

/*Terminal values*/
After a logarithmic transformation, put the value of the payoff into a vector P which will be used to save the option value.

/*Finite difference Cycle*/
At any time step, we have to solve the linear complementarity problem cf. there.

/*Init Control*/
We initializate the control pp.

/*Howard cycle*/

We solve the linear complementarity problem using the Howard algorithm, which consists in constructing a convergent sequence u^p whose limit is u.
Let epsilon>0 be given.

Step 1 Let u^k be given, we compute $i \to pp^k[i] = argmin(M^{pp}u^k(i) - f^{pp}[i])$ where $pp = 0$ or 1 (the domain is divided into 2 regions: the continuation region and the exercice region), M^0 is the matrix M^h issued from the discretization of the operator A, $M^1 = Id, f^0 = R, f^1 = Obst$.

Step 2 We solve the linear system $M^{pp^k}u = G^{pp^k}$ by the Gauss factorization. It gives u^{k+1}.

The stopping criteria is

$$\|u^{k+1} - u^k\|_\infty < epsilon. \tag{1}$$

/*Price*/

/*Delta*/

/*Memory Desallocation*/

References

[1] Howard, R.A.: Dynamic Programming and Markov Process. (MIT Press. 1960) 1