Callable Libor exotic products

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Remark: Most of what is presented here is taken from [Piterbarg 2005].

Callable Libor exotics are special interest rate products with callability characteristic. These derivatives, such as bermudan swaptions, callable capped floaters, callable inverse floaters etc, give the right to enter into various underlying interest rate instruments. The type of the underlying instrument decides the type of the derivatives, so we will start by presenting the different underlying instruments.

1 Callable Libor exotics

Let us consider a set of dates $T_0, T_1, ..., T_N$ with $0 = T_0 < T_1 < ... < T_N$ and $T_{i+1} - T_i = \delta_i$.

We note $F_i(t)$, for a certain date $t \leq T_i$, the value at date t of the Libor rate settled at T_i and paid at T_{i+1}.

By absence of arbitrage, the Libor rates are related to Zero Coupon bond curve $P(t,T)$ by :

$$F_i(t) = \frac{1}{\delta_i}\left(\frac{P(t,T_i)}{P(t,T_{i+1})} - 1\right)$$

The underlying instrument for a callable contract is a stream of payments $(X_i)_{1\leq i\leq N-1}$, each one fixed at T_i and paid T_{i+1}. A callable contract is a Bermuda option that gives the right to enter the underlying instrument on any date $(T_i)_{1\leq i\leq N-1}$, and if the option is exercised at T_n, the option ceases and the holder receives all the payments $(X_i)_{n\leq i\leq N-1}$, each one at its payment date T_{i+1}.

This stream of payments can be written as the difference between a coupon C_i and Libor rate F_i

$$X_i = \delta_i\ (C_i - F_i)$$

The exercise value is then equal to the value of all payments received after exercise date. Under the measure associated with the numeraire $B(t)$, the nth exercise value, denoted $E_n(t)$ is given by

$$E_n(t) = B(t)\sum_{i=n}^{N-1}\mathbb{E}_t\left[\frac{X_i}{B(T_{i+1})}\right]$$

So, if a callable Libor exotic is exercised at T_n, the holder receives $E_n(T_n)$ at T_n. But if he decides to not exercise, he holds a callable Libor exotic that has only the dates $(T_{n+1},...,T_{N-1})$ as exercise opportunities. We denote the value of this later contract $H_n(t)$.

Now, we will present next some examples of callable Libor exotics.

1.1 Bermudan swaption

This is the simplest contract. The underlying is a plain vanilla fixed-for-floating swap, where each coupon is of the form

$$C_i = c$$

where c is a fixed rate (called bermudan swaption strike).

1.2 Callable capped floater

For this contract, the coupon is a floating rate with a spread, capped from above

$$C_i = \min[F_i + s, c]$$

where s is the spread and c the cap.

1.3 Callable inverse floater

For this contract, the coupon is of the from

$$C_i = \min[\max[k - gF_i, f], c]$$

where k is the strike, g the gearing, f the floor and c is the cap of the inverse floating payment.

1.4 Callable range accrual

The coupon in this case is of digital-type

$$C_i = c\mathbb{1}_{\{F_i \in [l,b]\}}$$

Here c is the fixed rate for a range accrual payment, l is the lower range bound and b is the upper range bound.

1.5 Callable CMS spread

The payment stream of this product is linked to a spread between two different forward swap rates.

$$C_i = \max[\min[S_{i,l}(T_i) - S_{i,m}(T_i), c], f]$$

where $S_{i,l}(T_i)$ is $[T_i, T_l]$-forward swap rate, $S_{i,m}(T_i)$ is $[T_i, T_m]$-forward swap rate, c and f are a cap and a floor on the spread $S_{i,l}(T_i) - S_{i,m}(T_i)$ between the two CMS rates.

2 Valuing callable Libor exotics: regression methods

Callable Libor exotics are Bermuda-style options and can be valued using backward induction, with Monte Carlo techniques as proposed by [Longstaff and Schwartz 2001].

We note $Q(T_i)$ the price of the exotic product at time T_i, it can be constructed using the well known backward dynamic programming algorithm

$$\begin{cases} Q(T_{N-1}) & = & \max(0, E_{N-1}(T_{N-1})) \\ Q(T_n) & = & \max\left(E_n(T_n),\ H_n(T_n)\right)\ ,\text{ for } 1 \le n \le N-2 \\ Q(0) & = & \mathbb{E}_0\left[Q(T_1)\dfrac{B(T_0)}{B(T_1)}\right] \end{cases} \tag{1}$$

where the exercise value at the last date T_{N-1} is given by discounting the last payment X_{N-1} by Zero Coupon bond $P(T_{N-1}, T_N)$

$$E_{N-1}(T_{N-1}) = X_{N-1} P(T_{N-1}, T_N)$$

and the hold value $H_n(T_n)$ is induced from the option price at T_{n+1}

$$H_n(T_n) = \mathbb{E}_n \left[Q(T_{n+1}) \frac{B(T_n)}{B(T_{n+1})} \right]$$

These equations mean that at any exercise date T_n, the option holder has to decide wether to exercise the option and earn $E_n(T_n)$, or hold the option till the next exercise opportunity and in this case his portfolio is worth $H_n(T_n)$. Of course, the optimal decision depends of the maximum of the two values $\max\left(E_n(T_n),\ H_n(T_n)\right)$.

Now, to evaluate this option, i.e. to estimate $Q(0)$ using Monte Carlo simulations, we need to estimate the two conditional expectations

$$\begin{cases} E_n(T_n) & = & \mathbb{E}_n \left[\sum_{i=n}^{N-1} X_i \frac{B(T_n)}{B(T_{i+1})} \right] \\ H_n(T_n) & = & \mathbb{E}_n \left[Q(T_{n+1}) \frac{B(T_n)}{B(T_{n+1})} \right] \end{cases}$$

The Longstaff-Schwartz algorithm provides a way to estimate these conditional expectation and hence the optimal exercise strategy. In fact, we start by choosing random variables, called explanatory variables, $V(T_n)$, and two parametric families of real valued function $f_n(v;\alpha)$ and $g_n(v;\beta)$, where α and β are two vectors of parameters, then we choose special values of α and β that gives a good approximation of $E_n(T_n)$ and $H_n(T_n)$ in the sense:

$$\alpha_n = \underset{\alpha}{\operatorname{argmin}}\ \mathbb{E} \left[\left(Q(T_{n+1}) \frac{B(T_n)}{B(T_{n+1})} - f_n(V(T_n);\alpha) \right)^2 \right]$$

and

$$\beta_n = \underset{\beta}{\operatorname{argmin}}\ \mathbb{E} \left[\left(\sum_{i=n}^{N-1} X_i \frac{B(T_n)}{B(T_{i+1})} - g_n(V(T_n);\beta) \right)^2 \right]$$

then the equation (1) could approximated by

$$\begin{cases} Q(T_{N-1}) & = & \max(0, E_{N-1}(T_{N-1})) \\ Q(T_n) & = & \max\left(f_n(V(T_n);\alpha_n),\ g_n(V(T_n);\beta_n)\right)\ , \text{ for } 1 \le n \le N-2 \end{cases}$$

References

[Longstaff and Schwartz 2001] Longstaff, F., E. Schwartz (2001) Valuing American options by simulation: A simple least-squares approach. *Rev. Financial Stud*, 14 113–148. 3

[Piterbarg 2005] Vladimir V. Piterbarg (2005) Pricing and hedging callable Libor exotics in forward Libor models. *Journal of Computational Finance*, 8(2), 2005. 1