ap_out_laplace

Output parameters:

- Price
- Delta

Fixed Double Limit options are priced with Laplace Transform method of [1]

/*Computation of Laplace transform*/

$$\mathcal{L}(f(x)) = F(\lambda) = \int_0^\infty \exp(-\lambda x) f(x)\, dx$$

$\mu = \sqrt{2\lambda + \nu^2}$, $\nu = \frac{2y}{\sigma^2} - 1$, U =Upper_limit , L =Lower_limit , K =Strike , $x = S(0)$, $x^+ = S(0) * (1 + INC)$

$h = \frac{K}{x}$, $INC = 10^{-8}$

/*Call Case*/

We have (from [1])

$$F(\lambda) = \int_0^\infty \exp(-\lambda x) f(x)\, dx = \frac{\left(1-\left(\frac{x}{U}\right)^{2\mu}\right)}{\left(1-\left(\frac{L}{U}\right)^{2\mu}\right)} * \left\{ \left(\frac{L^2}{xK}\right)^{\mu} * \left(\frac{K}{x}\right)^{\nu+1} * \frac{1}{\mu(\mu-\nu)(\mu-\nu-1)} \right\}$$
$$+ \frac{\left(1-\left(\frac{L}{x}\right)^{2\mu}\right)}{\left(1-\left(\frac{L}{U}\right)^{2\mu}\right)} \left\{ 2\left(\frac{x}{U}\right)^{\mu-\nu-1} \left[\frac{1}{\mu^2-(\nu+1)^2} - \frac{\frac{K}{U}}{\mu^2-\nu^2}\right] + \left(\frac{xK}{U^2}\right)^{\mu} * \frac{\left(\frac{K}{x}\right)^{\nu+1}}{\mu(\mu+\nu)(\mu+\nu+1)} \right\}$$

/*Inversion parameters*/

According to the algorithm due to [2]

$A = 19.1$, $N = 15$, $M = 11$,

/* INVERSION */

We should remind that the inversion is made throw h.
We compute

$$sum = \frac{h}{e^{\frac{A}{2}}} * x = \frac{F_x(\frac{A}{2h})}{2} \quad \text{and } sum1 = \frac{h}{e^{\frac{A}{2}}} * x^+ = \frac{F_{x^+}(\frac{A}{2h})}{2}$$

/* Computation of $S[1] = s(N)$ and $Q[1] = s_{INC}(N)$ which is the fisrt approximation of $f(t)$ */

$$S\,[1] = \frac{h}{e^{\frac{A}{2}}} * s\,(N) = \frac{F_x(\frac{A}{2h})}{2} + \sum_{k=1}^{k=N} (-1)^k\, Re\left(F_x\left(\frac{A+2ik\pi}{2h}\right)\right)$$

$$Q\,[1] = \frac{h}{e^{\frac{A}{2}}} * s_{INC}\,(N) = \frac{F_{x^+}(\frac{A}{2h})}{2} + \sum_{k=1}^{k=N} (-1)^k\, Re\left(F_{x^+}\left(\frac{A+2ik\pi}{2h}\right)\right)$$

/* Computation of $s(N+j)$, $s_{INC}\,(N+j)$ $j <= M+1$ for Euler appromations */

$$S[j] = S[j-1] + (-1)^{N+j} * Re\left(F_x\left(\frac{A+2(N+j)k\pi}{2h}\right)\right);$$

$$Q[j] = Q[j-1] + (-1)^{N+j} * Re\left(F_{x^+}\left(\frac{A+2(N+j)k\pi}{2h}\right)\right);$$

/* Computation of Euler appromations */

$$Avg = Avg + Cnp(M,i) * s\,(N+i)\,;$$

$$Avg2 = Avg2 + Cnp(M,i) * \; s_{INC}\,(N+i)\,;$$

/* f(h) value */

$$Fun = \frac{e^{\frac{A}{2}}}{h} * 2^{-M} * Avg\ ;$$

$$Fun2 = \frac{e^{\frac{A}{2}}}{h} * 2^{-M} * Avg2\ ;$$

/*Black-Sholes price for call option*/
From this inversion, we can compute the Double-Limit call option, with the

help of the Black-Scholes price of a European call option

$dummy = Call\_BlackScholes\_73(1., h, t, r, divid, sigma, \&price, \&delta);$

$dummy = Call\_BlackScholes\_73(1., h2, t, r, divid, sigma, \&price2, \&delta2);$

/* Call Price */

$CTtK = x * price - x * exp(-r * t) * Fun;$
where the variable price is from

$Call\_BlackScholes\_73(1., h, t, r, divid, sigma, \&price, \&delta)$

/*Delta for call option*/

$\Delta_C = (CTtK - (price2 - price)/(h2 - h) * K)/x - exp(-r * t) * (Fun2 - Fun)/INC;$

/*Price*/

/*Delta */

References

[1] H.GEMAN M.YOR. Pricing and hedging double barrier options: a probabilistic approach. *Mathematical finance*, 6:365–378, 1996. 1

[2] J.ABATE W.WHITT. Numerical inversion of laplace transform of probability distribution. *ORSA Journal of Computing*, 7(1), Winter 1995. 2