

cf_exchange

Let

- T = maturity date ($T > t$)
- λ = exchange ratio
- $x1$ = spot1 price
- $x2$ = spot2 price
- t = pricing date
- $\sigma1$ = volatility1
- $\sigma2$ = volatility2
- ρ = correlation
- r = interest rate
- $\delta1$ = dividend1 yields
- $\delta2$ = dividend2 yields
- $\theta = T - t$

Here, closed formulas due to Johnson and Stulz are presented [1],[2].

We set

- $d = \frac{\log \frac{x_1}{x_2} + \left(\delta_2 - \delta_1 + \frac{\sigma_2^2}{2} \right) \theta}{\sigma \sqrt{\theta}}$
- $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$
- $d_i = \frac{\log \left(\frac{x_i}{K} \right) + \left(r - \delta_i + \frac{\sigma_i^2}{2} \right) \theta}{\sigma_i \sqrt{\theta}}, \quad i = 1, 2$
- $\rho_1 = \frac{\sigma_1 - \rho\sigma_2}{\sigma}$
- $\rho_2 = \frac{\sigma_2 - \rho\sigma_1}{\sigma}$

and M as the cumulative bivariate normal distribution function:

$$M(a, b; \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^a \int_{-\infty}^b e^{-\frac{x^2-2\rho xy+y^2}{2(1-\rho^2)}} dx dy.$$

Exchange Option

$$\text{PAYOFF} \quad E_T = (S_T^1 - \lambda S_T^2)_+$$

$$\text{PRICE} \quad E(t, x_1, x_2) = x_1 e^{-\delta_1 \theta} N(\hat{d}_1) - \lambda x_2 e^{-\delta_2 \theta} N(\hat{d}_2)$$

where

$$\hat{d}_1 = \frac{\log\left(\frac{x_1}{\lambda x_2}\right) + \left(\delta_2 - \delta_1 + \frac{\sigma^2}{2}\right)\theta}{\sigma\sqrt{\theta}}, \quad \hat{d}_2 = \hat{d}_1 - \sigma\sqrt{\theta}$$

References

- [1] H.JOHNSON. Options on the maximum of the minimum of several assets.
J.Of Finance and Quantitative Analysis, 22:227–283, 1987. [1](#)
- [2] R.STULZ. Options on the minimum or the maximum of two risky assets.
J. of Finance, 10:161–185, 1992. [1](#)