

fd_howard2d

Input parameters:

- SpaceStepNumber N
- TimeStepNumber M
- Epsilon

Output parameters:

- Price
- Delta1
- Delta2

The algorithm of Howard has been introduced by Howard in [\[1\]](#).

/*Memory Allocation*/

/*Covariance Matrix*/

/*Space localisation*/

Define the integration domain $D = [-l, l]^2$ using probabilistic estimation.

/*Space Step*/

Define the space step $h = \frac{2l}{N}$.

/*Time Step*/

Define the time step $k = \frac{T}{M}$.

/*Terminal Values*/

Put the value of the payoff into a vector P .

/*Homegenous Dirichlet Conditions*/

/*Factors of scheme*/

Initialize the matrix M^h issued from the discretization of the operator A in the case of Dirichlet Boundary conditions.

/*Finite difference Cycle*/

At any time step, we have to solve the linear complementarity problem.

/*Init Control*/

We initialize the control pp and the second member R .

/*Howard cycle*/

We solve the linear complementarity problem using the Howard algorithm, which consists in constructing a convergent sequence u^p whose limit is u .

Let $\epsilon > 0$ be given.

Step 1 Let u^k be given, we compute $(i, j) \rightarrow pp^k[i][j] = \operatorname{argmin}(M^{pp}u^k(i, j) - f^{pp}[i][j])$ where $pp = 0$ or 1 (the domain is divided into 2 regions: the continuation region and the exercise region), M^0 is the matrix M^h issued from the discretization of the operator A , $M^1 = Id$, $f^0 = R$, $f^1 = Obst$.

Step 2 We solve the linear system $M^{pp^k}u = G^{pp^k}$ by the Gauss factorization. It gives u^{k+1} .

The stopping criteria is

$$\|u^{k+1} - u^k\|_{\infty} < \epsilon. \quad (1)$$

/*Price*/

/*Delta*/

*/*Memory Desallocation*/*

References

- [1] Howard, R.A.: Dynamic Programming and Markov Process. (MIT Press. 1960) [1](#)