iCPPI with Gap Options under Kou model

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We present here the implementation of "Managing Gap Risks in iCPPI for life insurance companies: a risk/return/cost analysis" from the authors Kalife-Goudenège-Mouti (see [1]).

iCPPI model and implementation

The iCPPI is a product based on the Constant Proportion Portfolio Insurance (CPPI) named individualized CPPI. Consider an investment V_t where a proportion will be invested in a risky asset S_t and the remainder in a risk-free asset evolving with free rate r supposed to be constant. the insurer sets a number of initial parameter according to the insurer profile: an initial floor F_0 based on the guarantee G (which can be seen as the strike) and a multiplier m (greater than 1). At each moment the investment V_t (or portfolio) can be decomposed into the floor $F_t := F_0 e^{rt} := Ge^{-r(T-t)}$ and the cushion $C_t = V_t - F_t$.

Now the insurer invests mC_t in the risky asset (of course this quantity should be in $[0, V_t]$). In fact, as soon as the cushion vanishes the portfolio value falls below the floor, and no more risky exposure is allowed. All funds are invested in the risk-free asset in order to ensure the guarantee at maturity. In a continuous framework, there is closed formula for a Black-Scholes risky asset S_t and the portfolio never falls strictly below the floor. But in practice the CPPI is managed in discrete time at regular monitoring dates (say once a week) and we can fall strictly below the floor (loosing a percentage of the guarantee at maturity). We want to hedge this risk thanks to put options or other designed products. For this reason we consider a Levy dynamics (Kou model) for the risky asset in order to use the Gap Option as a hedging product of gap risk (i.e. when we falls below the floor).

So let S_t with Kou model dynamics be the risky asset, r the risk-free rate, T the maturity, m the multiplier, G the guarantee, V_0 the premium (i.e. the investment in the product), $(\sigma, \lambda, \lambda^+, \lambda^-, p)$ the classical parameters of the Kou model and N the number of monitoring dates.

At date t_n we invest mC_t in the risky asset, we simulate a scenario of the risky asset using Kou model till the next monitoring date t_{n+1}, we compute the cushion at time t_{n+1}, and loop over $n \in \{0, \dots, N-1\}$. Recursively we can write

$$C_{t_{n+1}} = \begin{cases} C_{t_n}\left(m\dfrac{S_{t_{n+1}}}{S_{t_n}} - (m-1)e^{rT/N}\right) & \text{if } C_{t_n} > 0 \\ \\ C_{t_n} e^{rT/N} & \text{if } C_{t_n} \leq 0 \end{cases}$$

We find $V_{t_{n+1}}$ through the relation $V_{t_{n+1}} = C_{t_{n+1}} + F_{t_{n+1}}$.

Looping over n gives the value of the product at maturity given a scenario (it is the output of the program).

Looping over the generation of scenario can give the distribution of the product at maturity. In a perfect monitoring, this maturity value should be greater than the guarantee (except that we sometimes break the floor).

Since we sometimes break the floor, we want to hedge this risk thanks to other product like put options or gap option. With a vanilla put option, it corresponds to buy put at each monitoring dates t_n with maturity t_{n+1} whose strike is $(1-1/m)e^{rT/N}S_{t_n}$. We need a number of mC_{t_n}/S_{t_n} puts. So the hedging cost is

$$\sum_n m\frac{C_{t_n}}{S_{t_n}} \times PUT_{Kou}(\text{asset} = S_{t_n}, \text{maturity} = t_{n+1}, \text{strike} = (1-1/m)e^{rT/N}S_{t_n}).$$

With gap options it suffices to buy one Gap Option with level $1/m$ so the hedging cost is

$$\sum_n GAP_{Kou}(\text{asset} = S_{t_n}, \text{maturity} = t_{n+1}, \text{level} = 1/m).$$

In the program, there is a supplementary parameter which is the number of points for the generation of the Levy dynamics. Its default value is 100 which seems sufficient between two monitoring dates.

Conclusion

The model is relatively simple to understand and easy to simulate. The cost of the hedging is a good information for the insurer and is relatively easy to compute. A complete study of the different hedging is given in "Managing Gap Risks in iCPPI for life insurance companies: a risk/return/cost analysis" from the authors Kalife-Goudenège-Mouti [1].

References

[1] A.Kalife S.Mouti L.Goudenege. Managing gap risks in icppi for life insurance companies: A risk/return/cost analysis. *Insurance Markets and Companies: Analyses and Actuarial Computations*, 2, 2014. 1, 2