ap_fixedasian_laplace

Output parameters:

- Price
- Delta

Fixed Asian options are priced with Laplace Transform method of [1] and [2]

/*Computation of Laplace transform*/

$\mathcal{L}(f(x)) = F(\lambda) = \int_0^\infty \exp(-\lambda x) f(x)\, dx = \frac{\int_0^{\frac{1}{2q}} \exp(-u)(1-2qu)^{\left(\frac{\mu+\nu}{2}+1\right)} u^{\left(\frac{\mu-\nu}{2}-2\right)} du}{\lambda(\lambda-2-2\nu)\Gamma\left(\frac{\mu-\nu}{2}-1\right)}$,
$\mu = \sqrt{2\lambda+\nu^2}$, $q = \frac{\sigma^2}{4S(t)}\{k*(T-t)\}$, $\nu = \frac{2y}{\sigma^2}-1$, $S_{INC}(t) = S(t)(1+INC)$
$INC = 10^{-8}$, $p = q*\frac{1}{1+INC}$

/* Integral Computation */

This formula is from [2]

$\int_0^{\frac{1}{2q}} \exp(-u)(1-2qu)^{\left(\frac{\mu+\nu}{2}+1\right)} u^{\left(\frac{\mu-\nu}{2}-2\right)} du$

/* Rieman sums */

Here, we compute the integral with the rieman sum.

$\sum_{j=1}^{j=999} \frac{1}{1000} * \exp\left(-\frac{u_j}{2q}\right)(1-u_j)^{\left(\frac{\mu+\nu}{2}+1\right)} \left(\frac{u_j}{2q}\right)^{\left(\frac{\mu-\nu}{2}-2\right)}$

$u_j = j*\left(\frac{1}{1000}\right)$.

$$\Theta_q(\lambda)=\frac{\sum_{j=1}^{j=999}\frac{1}{1000}*\exp\left(-\frac{u_j}{2q}\right)(1-u_j)^{\left(\frac{\mu+\nu}{2}+1\right)}\left(\frac{u_j}{2q}\right)^{\left(\frac{\mu-\nu}{2}-2\right)}}{\lambda(\lambda-2-2\nu)\Gamma\left(\frac{\mu-\nu}{2}-1\right)}$$

/*Inversion parameters*/

Using the algorithm [3]

$A=19.1,\ N=15,\ \ M=11,$

/* INVERSION */

We should remind that the inversion is made throw h.

We compute $sum=\frac{h}{e^{\frac{A}{2}}}*s\left(t\right)=\frac{F_q\left(\frac{A}{2h}\right)}{2}$ and $sum1=\frac{h}{e^{\frac{A}{2}}}*s_{INC}\left(t\right)=\frac{F_p\left(\frac{A}{2h}\right)}{2}$

/* Computation of $S[1]=s(N)$ and $Q[1]=s_{INC}(N)$ which approximate $f(t)$ */

$S\left[1\right]=\frac{h}{e^{\frac{A}{2}}}*s\left(N\right)=\frac{F_q\left(\frac{A}{2h}\right)}{2}+\sum_{k=1}^{k=N}\left(-1\right)^k Re\left(F_q\left(\frac{A+2ik\pi}{2h}\right)\right)$

$Q\left[1\right]=\frac{h}{e^{\frac{A}{2}}}*s_{INC}\left(N\right)=\frac{F_p\left(\frac{A}{2h}\right)}{2}+\sum_{k=1}^{k=N}\left(-1\right)^k Re\left(F_p\left(\frac{A+2ik\pi}{2h}\right)\right)$

/* Computation of $s(N+j),\ \ s_{INC}\left(N+j\right)\ \ j<=M+1$ for Euler appromations */

$S[j]=S[j-1]+\left(-1\right)^{N+j}*Re\left(F_q\left(\frac{A+2(N+j)k\pi}{2h}\right)\right);$

$Q[j]=Q[j-1]+\left(-1\right)^{N+j}*Re\left(F_p\left(\frac{A+2(N+j)k\pi}{2h}\right)\right);$

/* Computation of Euler appromations */

$Avg=Avg+Cnp(M,i)*s\left(N+i\right);$

$Avg1=Avg1+Cnp(M,i)*\ s_{INC}\left(N+i\right);$

/* f(h) value */

Then we have the value of the inversion of the Laplace Transform.

$Fun = \frac{e^{\frac{A}{2}}}{h} * 2^{-M} * Avg$;

$Fun1 = \frac{e^{\frac{A}{2}}}{h} * 2^{-M} * Avg1$;

/* Call Price */

Taking the Call price formula from [2]

$C_{T,t}\left(K\right) = \frac{\exp(-r*(T-t))*4.0*S(t)}{(T-t)\sigma^2} C\left(h,q\right)$

/* Put Price from Parity*/

Simple calculuous give the call-put parity relationship

$P_{T,t}\left(K\right) = C_{T,t}\left(K\right) - K * \exp\left(-r*\left(T-t\right)\right) - S\left(t\right) * \exp\left(-r*\left(T-t\right)\right) * \left(\exp\left(-\left(r-divid\right)*\left(T-t\right)\right) - 1\right) * \frac{1}{(T-t)*(r-divid)}$

/*Delta for call option*/

Here we derive the formula from [2] with respect to the variable $S(t)$

$\Delta_C = \left(\frac{\exp(-r*(T-t))*4.0*S_{INC}(t)}{(T-t)\sigma^2} C\left(h,p\right) - \frac{\exp(-r*(T-t))*4.0*S(t)}{(T-t)\sigma^2} C\left(h,q\right)\right) * \frac{1}{S(t)*INC}$

/*Delta for put option*/

We use again the call-put parity relation

$\Delta_P = \Delta_C - \exp\left(-r*\left(T-t\right)\right) * \left(\exp\left(-\left(r-divid\right)*\left(T-t\right)\right) - 1\right) * \frac{1}{(T-t)*(r-divid)}$

/*Price*/

/*Delta */

References

[1] M.YOR. On some exponiancial functionals of brownian motion. *Adv. Appl. Pro.*, 24:509–531, 1992. 1

[2] H.GEMAN M.YOR. Besssel processes, asian options, and perpetuities. *Mathematical finance*, 3:349–375, 1993. 1, 3

[3] J.ABATE W.WHITT. Numerical inversion of laplace transform of probability distribution. *ORSA Journal of Computing*, 7(1), Winter 1995. 2