cf_call_merton

Let

- $T =$ maturity date $\quad (T > t)$
- $K =$ strike price
- $x =$ spot price
- $t =$ pricing date
- $\sigma =$ volatility
- $r =$ interest rate
- $\delta =$ dividend yields
- $\lambda =$ jump intensity
- $\kappa = \mathbf{E}U - 1$ where $\mathbf{E}U$ is the expectation over the random jump variable U
- $\theta = T - t$

We assume [1] that the jump variable U is log-normally distributed with constant mean and variance. Specifically, let $\mathbf{E}[\ln U] = \mu$ and $\mathbf{E}[(\ln U)^2] = \mu^2 + v$, such that $\mathbf{E}U = e^{\mu + \frac{v}{2}}$.

Set:

$$d_1 = \frac{\log\left(\frac{x}{K}\right) + \left(r_n + \frac{\sigma_n^2}{2}\right)\theta}{\sigma_n\sqrt{\theta}} \qquad d_2 = d_1 - \sigma_n\sqrt{\theta},$$

where

$$\sigma_n^2 = \sigma^2 + \frac{nv}{\theta} \qquad r_n = r - \delta - \lambda\kappa + \frac{n}{\theta}\ln(1+\kappa).$$

and N as the cumulative normal distribution function:

$$N(d) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d} e^{-x^2/2}dx.$$

Call Option

PAYOFF	$C_T = (S_T - K)_+$
PRICE	$C(t,x;K) = \sum_{n=0}^{\infty} \frac{(\lambda(1+\kappa)\theta)^n}{n!} e^{-\lambda(1+\kappa)\theta - \delta\theta}[xN(d_1) - Ke^{-r_n\theta}N(d_2)]$
DELTA	$\dfrac{\partial C(t,x;K)}{\partial x} = \displaystyle\sum_{n=0}^{\infty} \frac{(\lambda(1+\kappa)\theta)^n}{n!} e^{-\lambda(1+\kappa)\theta - \delta\theta} N(d_1)$

References

[1] R.C.MERTON. Option pricing when the underlying stocks returns are discontinuous. *Journ. Financ. Econ.*, 5:125–144, 1976. 1