

cf_digit

Let

- T = maturity date ($T > t$)
- K = strike price
- x = spot price
- t = pricing date
- σ = volatility
- r = interest rate
- δ = dividend yields
- $\theta = T - t$
- $b = r - \delta$

Set:

$$d_1 = \frac{\log\left(\frac{x}{K}\right) + \left(b + \frac{\sigma^2}{2}\right)\theta}{\sigma\sqrt{\theta}} \quad d_2 = d_1 - \sigma\sqrt{\theta}$$

and N as the cumulative normal distribution function:

$$N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^d e^{-x^2/2} dx.$$

Digit Option

$$\text{PAYOFF :} \quad C_T = \begin{cases} K & \text{if } S_T > K, \\ 0 & \text{otherwise} \end{cases}$$

$$\text{PRICE :} \quad C(t, x) = K e^{-r\theta} N(d_1)$$

$$\text{DELTA :} \quad \frac{\partial C(t, x)}{\partial x} = K \frac{e^{-d_1^2/2}}{\sqrt{2\pi\theta}\sigma x}$$