

## fd\_explicit\_cir1d\_capfloor

Input parameters:

- Time StepNumber  $M$

Output parameters:

- Price

The stochastic differential equation representing the short rate is given by

$$dr_t = k(\theta - r_t)dt + \sigma \sqrt{r_t}dW(t)$$

The price of the zero-coupon bond is solution of the following PDE

$$u_t + \frac{1}{2}\sigma^2 r u_{rr} + [k(\theta - r)]u_r - ru = 0, u(r, T, T) = 1$$

that we solve using explicit scheme of Hull-White[1]. The price of the option is obtained solving the same PDE with boundary condition at the maturity of the option  $T$ , the price of the Zero Coupon Bond. The pricing procedure is in two steps: in the first step (INITPROBA) we compute probabilities associated to the explicit scheme; this is done simply matching the first and the second moment of the change in  $r$  over time step  $\Delta t$ . The branching in the lattice is modified at boundary points  $r = r_{min}$  and  $r = r_{max}$  to ensure that the probabilities associated with all three branches remain positive. For this purpose Hull-White[1] propose alternative branching procedures in the explicit finite difference method.

The second step is standard dynamic programming backward pricing algorithm. A cap(floor) is equivalent to a portfolio of European zero-coupon Put(Call)-Options.

## References

- [1] J.Hull and A.WHITE. Valuing derivative securities using the explicit finite difference method. *Journal of Financial and Quantitative Analysis*, 25:87–100, 1990. [1](#)