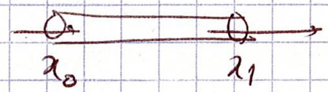
Modèle physiques cf. Strauss p13+–

Diffusion



Concentration "substance" à travers un fluide au repos.

Loi de Fick. Mvt de + haute concentraO vers + faible conce.

Vitesse $\propto$ gradient concentration

$$M(t) = \int_{x_0}^{x_1} u(x,t)\,dx \longrightarrow \frac{dM}{dt} = \int_{x_0}^{x_1} \partial_t u(x,t)\,dt$$

Conservation masse : Chgt de M = flux entrant – flux sortant

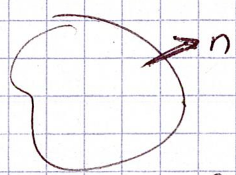
Flux en x_0 : $q(t) = -k\,\partial_x u(x_0,t)$

x_1 $\quad q(t) = +k\,\partial_x u(x_1,t)$

$$\frac{dM}{dt} = \int_{x_0}^{x_1} \partial_t u(x,t)\,dx = k\,\partial_x u(x_1,t) - k\,\partial_x u(x_0,t)$$

$$\Rightarrow \boxed{\partial_t u - k\,\partial_{xx} u = 0}$$

2D/3D



Flux $= k\,\nabla u \cdot n \;(= -\vec{q}\cdot\vec{n})$ $\qquad q = -k\nabla u$

$$\int_D \frac{dM}{dt} = -\int_{\partial D} +\vec{q}\,\vec{n} = \int k\nabla u\, n \longrightarrow \text{th divergence (Green)}$$

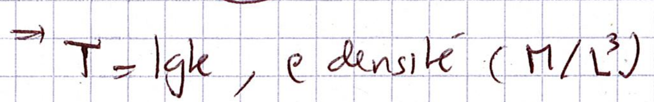
$$\Rightarrow \frac{\partial u}{\partial t} - k\Delta u = 0$$

$$\int_D \operatorname{div}\vec{q} = \int_{\partial D} \vec{q}\,\vec{n}$$

Corde vibrante

Corde (= 1D), élastique, long L, vibrations transverse dans un plan

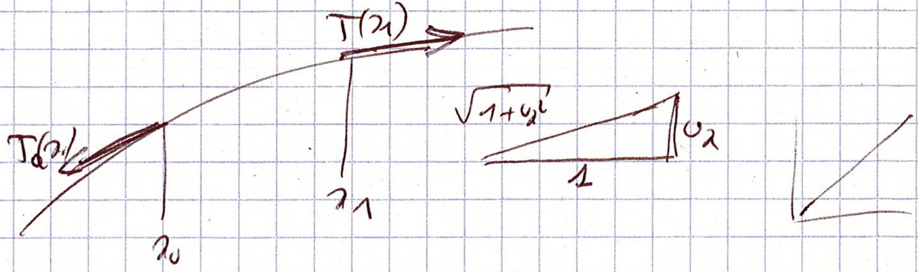
Deplact $u(x,t)$



Corde flexible $\Rightarrow$ T = lgle, ρ densité (M/L^3)

Newton entre x_0, x_1

Fig3



Newton $F = m\gamma$

Longitudinal. $\left.\frac{T}{\sqrt{1+u_x^2}}\right|_{x_0}^{x_1} = 0 \Rightarrow \underline{\underline{T = cte}}$

cos

$\left.\frac{T u_x}{\sqrt{1+u_x^2}}\right|_{x_0}^{x_1}$ sin $= \int_{x_0}^{x_1} \rho\, \partial_t^2 u \, dx$

Petit mvt $\Rightarrow 1+u_x^2 \simeq 1$

$(Tu_x)_x = \rho u_{tt} \Rightarrow \frac{1}{c^2} u_{tt} - u_{xx} = 0$

$c = \sqrt{\frac{T}{\rho}}$ équation des ondes

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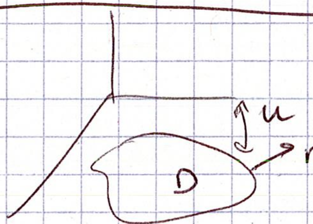
dim 2/3



$F = \int_{\partial\Omega} T\frac{\partial u}{\partial n}$ contact normal $= \int \rho u_{tt} = m\gamma$

+ th divergence

$\rho\partial_t^2 u = \nabla\cdot(T\nabla u) \Rightarrow \frac{1}{c^2}\partial_t^2 u - \Delta u = 0$

Equilibre $\Rightarrow -\Delta u = 0$ (ou f) si force ⇊

Classification

$$a\partial_x^2 u + 2b\partial_{xy}^2 u + c\partial_y^2 u + d\partial_x u + e\partial_y u + f\cancel{u} = 0$$

Eq lin + générale du 2nd ordre

$\boxed{\text{Lien avec les "coniques"}}$

$$ax^2 + 2bxy + cy^2 + dx + ey + f = 0$$

$$\underbrace{\binom{x}{y}^T}_{X} A \underbrace{\binom{x}{y}}_{Y} + \binom{x}{y}^T h + f = 0$$

① A inversible $\exists X_0 \quad AX_0 = -h \qquad Y = X - X_0$

$$\cancel{X^T A(X - X_0) + f = 0} \qquad Y^T A Y + g = 0 \qquad g = X_0^T A X_0\, f$$

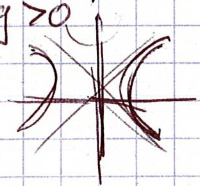
~~$g<0 \to \phi$, $g = 0$ 2 dtes, $g>0$~~

diagonaliser A $\quad A = Q^T \Lambda Q \quad \Lambda = \begin{pmatrix} \lambda_1 & \\ & \lambda_2 \end{pmatrix} \quad Z = QY$

$$\boxed{\lambda_1 z_1^2 + \lambda_2 z_2^2 = g}$$

λ_1, λ_2 signes $\neq$ $\quad \lambda_1 > 0, \lambda_2 < 0 \qquad \alpha^2 z_1^2 - \beta^2 z_2^2 = g \qquad g>0$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{Hyp}$$



λ_1, λ_2 m̂ signe, g signe contraire $\qquad g = 0 \to 2$ dtes

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{ellipse}$$

② A non inversible $\quad A = Q^T \Lambda Q \quad \Lambda = \begin{pmatrix} \alpha & 0 \\ 0 & 0 \end{pmatrix}, \quad Y = QX \quad \alpha \neq 0$

$$\alpha y_1^2 + u y_1 + v y_2 + f = 0 \qquad z_1 = y_1 + \mu$$

$$z_2 = -g/\mu$$

$$\cancel{\alpha z_1^2 + \mu z_1 + g = 0}$$

$v \neq 0$ sinon 2 dtes;

~~$z_2 = y_1 + \frac{f}{v}$~~ $\quad z_1 = y_1 + \lambda, \quad z_2 = y_2 + \mu \quad$ tq $(0,0) \in$ ense.

$$y_2^2 - 2p y_1^2 = 0 \to \underline{y^2 = 2px}$$

~~VP~~ $A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ $\Delta = ac - b^2$

$\begin{cases} \Delta < 0 \rightarrow & \text{ellipse} \\ \Delta > 0 & \text{hyperbole} \\ \Delta = 0 & \text{parabole} \end{cases}$

EDP 2ordre $a \partial_x^2 u + 2b \partial_{xy} u + c \partial_y^2 u = 0$

$\Delta < 0$ elliptique $\rightarrow$ $\partial_x^2 u + \partial_y^2 u = 0$ Laplace

$\Delta > 0$ hyperb $\partial_x^2 u - \partial_y^2 u = 0$ Ondes

$\Delta = 0$ paraboliq $\partial_{\cancel{y}}^2$ $= 0 \rightarrow$ Chaleur (1'ordre)